Detection and Comparison of Objects Using Two Dimensional Geometric Moment Invariants

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Abstract—**In this paper, we present the concept of Moment Invariants. Objects can be detected with the help of Moment Invariants. Different Moment Invariants are discussed in this paper. The pitfalls of Fourier-Mellin Invariants and Legendre Moment Invariants are discussed. Two Dimensional Moments of N × N images are sampled and proved that 2D Geometrical Moment Invariants are far better than the other Invariants in terms of the object detection.**

Index Terms—**Image function, algebraic invariants, polar coordinates, affine groups, information redundancy, fourier-mellin invariants, legendre moment invariants, counter-based shape descriptors, geometric moment invariants.**

I. Introduction

Moment Invariants are commonly used for identification and inspection of shapes of images. Moment Invariants plays a vital role in many applications of computer vision and pattern recognition. The basic idea of Moment Invariants is to describe the objects by a group of features which provide discrimination power to identify objects from different groups. The 2D Moment Invariants were firstly introduced by Hu [1] in 1962. Who used the results of theory of algebraic invariants and derived seven known invariants to rotation of 2D objects. Dudani [2] and Belkasim [3] explained their work on applications of aircraft silhouette recognition. Li [4] and Wong [5] exploded the invariants order to fine and fire respectively. Flusser and Suk[6] employed moment invariants in template matching and recognition of satellite images. Wang [7] proposed illumination invariants suitable for feature classification. Resis [8] revised some of the geometrical proofs of Hu. Teague [9] proposed the use of orthogonal moments based on the history of orthogonal polynomials. Different types of Image Moments, like Geometrical Moments, Legendre Moments, Zernike Moments, Fourier- Mellin Moments, Pseudo-Zernike Moments, Complex Moments were developed based on the capability of image description, noise sensitivity etc., Wallin [10] discussed an algorithm for the formation of moment invariants of any order. Flusser [11] discussed the independence of Two Dimensional Rotational Moments via Complex Moments. He also constructed 2D moment invariants basis and its independence and completeness. Van Gool [12] introduced affine-photo metric invariants of gray level and color images. Flusser and Suk [13] derived invariants to convolution which an arbitrary PSF. But these descriptions were invariants to translation only.

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II. The Concept of Moment Invariants

Moment invariants were first introduced by the Hu. By image functions we understand any real function $\mathrm{f(x,y)} \in Li$ having a bounded support and a non-zero integral. Translation and scale variance of a dimension moment invariants are easy to be eliminated. Let $m_{p_1 \dots p_n}$ represent the n-dimension $(p_1 + \cdots + p_n)^{\text{th}}$ order moment of a piecewise continuous density function $h(x_1, \dots, x_n)$, it can be defined as

$$m_{p_1 \dots p_n} = \int \dots \int_{\infty} x_1^{p1} \dots\dots x_n^{pn} h(x_1, \dots, x_n) dx_1, \dots, dx_n \quad (1)$$

The scale variants can be eliminated based on the concept of algebraic invariants [14] as follows.

$$\eta_{p_1 \dots p_n} = \frac{\mu_{p1 \dots pn}}{\mu_{0 \dots 0}{}^{(p1+\cdots+pn)/n+1}} \quad (2)$$

A. Expression for Moment-Based Rotation Invariants

Rotation invariant is achieved by using Zernike's moment invariants, wavelet moment invariants and Li's moment invariants.

Let $f(x, y)$ represent a two dimensional binary image object in the (x, y) coordinate and Let $f(r, \theta)$ be its corresponding polar coordinate. Then the relationship between $f(x, y)$ and $f(r, \theta)$ is specified as

$$x = r\cos(\theta),$$

$$y = r\sin(\theta).$$

To derive rotation invariant moments, the following general expression is used

$$f_{pq} = \iint f(r, \theta)\, g_p(r) e^{jq\theta}\, r\, dr\, d\theta$$

where f_{pq} is the pq order moment, $g_p(r)$ is a function of radial variable r, and P and q are integer parameters.

III. Fourier-Mellin Invariants

The optical research community was introduced the Fourier-Mellin transform in 1970s. These transforms are used now-a-days in Digital Image and Signal Processing. The main concept of Fourier-Mellin transform is with the study of similarity transformations. Ghorbel [15] work focused on the Fourier transform defined on 2D and 3D parameterization. Tursci [16] explained some Fourier transforms for sub groups

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of the affine group.

Let f denote a function representing a gray level image defined over a compact set of $\mathbb{R}^2$. The standard Fourier-Mellin Transform of f is given by

$$\forall(k,v) \in \mathbb{Z} \times \mathbb{R}, \mu_f(k,v) = \frac{1}{2\pi}\int_0^\infty \int_0^{2\pi} f(r,\theta) r^{-ir} e^{-ik\theta}\, d\theta \frac{dr}{r} \quad (3)$$

f Is assumed to be Summable over $\mathbb{R}_+^* \times S^1$ under the measure $\frac{dr}{r}$, ie

$$\int_0^\infty \int_0^{2\pi} \left|f(r,\theta) r^{-ir} e^{-ik\theta}\right| d\theta \frac{dr}{r} = \int_0^\infty \int_0^{2\pi} \frac{1}{r} f(r,\theta) d\theta\, dr \quad < \infty \quad (4)$$

IV. Legendre Moments

Legendre moments were introduced by Teague [17]. Tel and chain [18] stated that orthogonal Legendre moments can be used to represent an image with a minimum amount of information redundancy.

The two dimensional Legendre moments of order $(p+q)$ for image intensity function $f(x,y)$ are defined as

$$L_{pq} = \frac{(2p+1)(2q+1)}{4} \int_{-1}^{1} \int_{-1}^{1} p_p(x) p_q(y) f(x,y) dx\, dy \quad (5)$$

where $p_p(x)$ is the pth order Legendre polynomial Legendre descriptors of the third and fourth order are used to test the invariance.

Third order descriptions are

$$\phi_1 = \frac{35}{8}\eta_{3,0}$$

$$\phi_2 = \frac{45}{8}\eta_{2,1}$$

$$\phi_3 = \frac{45}{8}\eta_{1,2}$$

$$\phi_4 = \frac{35}{8}\eta_{0,3}$$

Fourth order descriptions are

$$\phi_5 = \frac{9}{4}\left[\frac{35}{8}\eta_{4,0} - \frac{30}{8}\eta_{2,0} + \frac{3}{8}\eta_{0,0}\right]$$

$$\phi_6 = \frac{105}{8}\eta_{3,1}$$

$$\phi_7 = \frac{25}{4}\left[\frac{9}{4}\eta_{2,2} - \frac{3}{4}\eta_{2,0} - \frac{3}{4}\eta_{0,2} + \frac{1}{4}\eta_{0,0}\right]$$

$$\phi_8 = \frac{105}{8}\eta_{1,3}$$

$$\phi_9 = \frac{9}{4}\left[\frac{35}{8}\eta_{0,4} - \frac{30}{8}\eta_{0,2} + \frac{3}{8}\eta_{0,0}\right]$$

V. Proposed Work

The proposed work in this paper used geometric moment invariants and formed to be superior compare Fourier-Mellin invariants and Legendre moments.

A. Geometric Moment Invariants

The Geometric Moment Invariants produce a group of features vectors that are invariants under shifting, rotation and scaling. Regular moment invariants are most popular counter-based shape descriptions derived by Hu. Geometric moment invariants were first introduced by Hu, which are derived from the theory of algebraic invariants.

Geometric moment were successfully applied on the alphabet A shown in different shapes. The alphabet image A is used to get the range of invariant. The comparison of invariants feature vectors are shown in the table.

Two dimensional moments of a digitally sampled $N \times N$ Image has gray function $f(x,y)$, $(x,y = 0 \ldots..N-1)$

And is given by

$$m_{pq} = \sum_{x=0}^{x=N-1} \sum_{y=0}^{y=N-1} (x)^p (y)^q f(x,y) \quad (6)$$

where $p,q = 0,1,2 \ldots..$

The moments $f(x,y)$ translated by an amount (a,b) are defined as

$$\mu_{pq} = \sum_x \sum_y (x+a)^p (y+b)^q f(x,y) \quad (7)$$

VI. Results

The alphabet A in different shapes (I1, I2, I3 etc.,) has been adopted as the text image and the simulate results of average invariants ϕ_1, ϕ_2, ϕ_3 and ϕ_4 are compared with Fourier-Mellin, Legendre and Geometric Invariants as shown in table 1.

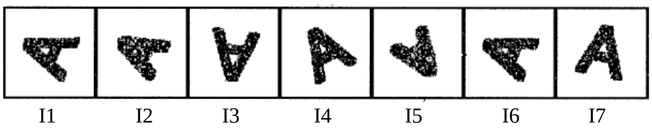

Fig. 1. Test images of the alphabet A

TABLE I: Invariants Features for Comparison

Image	Fourier- Mellin	Legendre	Geometrical
I1	0.16134	0.182431	0.24321
I2	0.19169	0.05681	0.19928
I3	0.23912	0.22141	0.26815
I4	0.278236	0.18149	0.28816
I5	0.284915	0.28551	0.29912
I6	0.205873	0.22121	0.26124
I7	0.351478	0.32114	0.368194

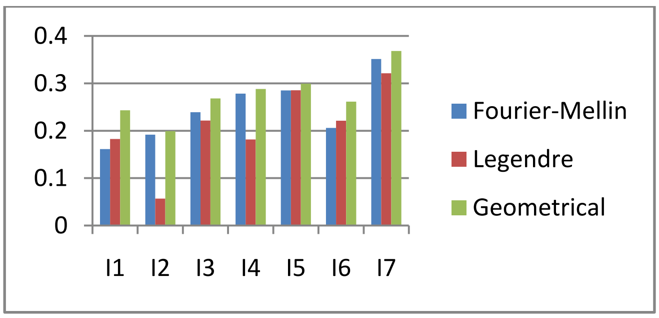

Fig.2. Performance comparison of fourier and legendre moment invariants with geometrical moment invariants

Result shows that the Geometric Moment Invariants are more superior in all aspects. The first order Invariants of Geometric Moments are compared with Fourier and Legendre Moments and are shown with bar chart.

VII. Conclusion

Different Moment Invariants are compared. The first order

Invariants of Fourier, Legendre and Geometric Moment Invariants comparison shows that Geometric Moment's Invariants are best suitable for object detection.

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