The Weighting Analysis of TOPSIS via Grey System Theory – An Example in School Exam's Score

Chien-Tung Chen, Mei-Li You, and Kun-Li Wen

***Abstract*—In the mathematics model of weighting analysis filed, due to the essential of weighting is quite subjective. Hence, the paper presents an objective weighting decision method, which can reach objective weighting and make the weighting given into an objective state to reduce the subjectively. Firstly, the mathematics model of Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is introduced, and point out the imperfection, which is the subjective of weighting. Secondly, the objective weighting analysis model of globalization grey relational grade (GGRG) in grey system theory is presented to transfer the subjective state into objective state. Thirdly, an example in education is given to verify our new approach. In addition, the article also uses Matlab toolbox to make the calculation procedures more accurately and more quickly. As a result, it is hoped that through this method, the purpose of integrating computer and the rational of weighting can be achieved.**

***Index Terms*—Weighting, TOPSIS, GGRG, Matlab.**

I. Introduction

In the past, there are many related methods of weighting analysis, including AHP, factor analysis, ISM, Fuzzy method, grey system theory and so on [1]-[5], However, the weighting is very subjective, one of the weighting analysis in soft computing [6], which is called technique for order preference by similarity to ideal solution(TOPSIS) also have the same problem [7]. Therefore, the paper presented a objective weighting analysis method, which is called globalization grey relational grade (GGRG) to make the weighting can transfer from subjective into objective [8]. Also according to the mentioned above, we can find that the calculation steps of previous are very complexity, hence, it is necessary to have software to assist the analysis and calculate for huge numbers [9]. Hence, the paper use Matlab to develop the GGRG-TOPSIS toolbox [10], which not only assists a huge number of numerical calculations, but also enhances the breadth and practicality of the weighting analysis in the field of hierarchy relationship.

The section II of this study are the mathematical model of TOPSIS and GGRG, mainly explains the analysis steps of our research, Section III is the real example in the student exam, where actual data was substituted into the mathematical model to derive the results, and the development Matlab toolbox. The final section of this study consists of a conclusion and recommendations for future research.

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II. Mathematics Model

A. Technique for Order Preference by Similarity to Ideal Solution

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), which is proposed by Hwang and Yoon, is often used to solve multi-attribute decision-making problems [7]. This method assumes that each evaluation indicator has a monotonically increasing or monotonically decreasing feature. Among them, the so-called positive ideal solution is composed by the optimal value of all indicators. Conversely, the negative ideal solution is the worst value of all the component indicators. The selection scheme is calculated by Euclidean distance, and the main concept is to evaluate the comparison of indicator to ideal solution. The best selected solution should have the shortest distance to positive ideal solution and the longest distance to negative ideal solution.

Pi points out that the order achieved from this method can avoid the error of having the shortest distance to both positive ideal solution and negative solution. Also, it can avoid the error of having the longest distance to positive ideal solution and negative ideal solution. It can avoid arise the disadvantage of comparison difficulty [11]. There are total seven steps of the calculation of order preference by similarity to ideal solution method and they are explained as follows [12].

Input the project's data

$$D = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \cdots & x_{mn} \end{bmatrix} \tag{1}$$

Use equation (2) to normalize the data in equation (1)

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{2}$$

the normalization matrix is shown in equation (3)

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$$R=\begin{bmatrix} r_{11} & r_{12} & r_{13} & \cdots & r_{1n} \\ r_{21} & r_{22} & r_{23} & \cdots & r_{2n} \\ r_{31} & r_{32} & r_{33} & \cdots & r_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & r_{m3} & \cdots & r_{mn} \end{bmatrix} \tag{3}$$

Decide the weighting of $[\omega_1,\omega_2,\omega_3,\cdots,\omega_n]$: Objective and subjective.

Calculate the weighting decision matrix

$$V=\begin{bmatrix} \omega_1 r_{11} & \omega_2 r_{12} & \omega_3 r_{13} & \cdots & \omega_n r_{1n} \\ \omega_1 r_{21} & \omega_2 r_{22} & \omega_3 r_{23} & \cdots & \omega_n r_{2n} \\ \omega_1 r_{31} & \omega_2 r_{32} & \omega_3 r_{33} & \cdots & \omega_n r_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega_1 r_{m1} & \omega_2 r_{m2} & \omega_3 r_{m3} & \cdots & \omega_n r_{mn} \end{bmatrix}$$

$$=\begin{bmatrix} v_{11} & v_{12} & v_{13} & \cdots & v_{1n} \\ v_{21} & v_{22} & v_{23} & \cdots & v_{2n} \\ v_{31} & v_{32} & v_{33} & \cdots & v_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ v_{m1} & v_{m2} & v_{m3} & \cdots & v_{mn} \end{bmatrix} \tag{4}$$

Calculate the positive ideal solution A^+ and ideal negative solution A^-

$$A^+ = \max.\{v_i^+\} = (v_1^+, v_2^+, v_3^+, \cdots, v_m^+)$$

$$A^- = \min.\{v_i^-\} = (v_1^-, v_2^-, v_3^-, \cdots, v_m^-) \tag{5}$$

Calculate the positive ideal distance S_i^+ and negative ideal distance S_i^-

$$S_i^+ = \sqrt{\sum_{j=1}^{n}(v_{ij}-v_j^+)^2},\ S_i^- = \sqrt{\sum_{j=1}^{n}(v_{ij}-v_j^-)^2} \tag{6}$$

$i=1,\ 2,\ 3,\ \cdots,\ m$

Calculate the relative approaching of ideal distance C_j, then, the weighting can be found.

$$C_j = \frac{S_i^-}{S_i^+ + S_i^-},\quad i=1,\ 2,\ 3,\ \cdots,\ n,\ j=1,\ 2,\ 3,\cdots,\ n \tag{7}$$

In the mentioned above, we can find in step 3, it exists a subjective part in the traditional TOPSIS, the weighting's decision is subjective, and different subjective value cause different results. Hence, the objective weighting find method is proposed in next section.

B. Grey Relational Grade

The mathematical foundation of grey relational grade can be described as follows [8]

Factor space: Assume $P(X)$ is one theme and Q is one relationship. If a characteristic exists with key factors, such as: countable intention factor, expansion of factor and independence factor for the combination of { $P(X)\,;Q$ }, { $P(X)\,;Q$ }, then it can be called a factor space [11].

The comparison of sequence: Assume a sequence as

$$x_i(k) = (x_1(k), x_2(k)\cdots,\cdots, x_n(k)) \tag{8}$$

$$k=1,2,3,\ldots n\in N,\ i=1,2,3,\ldots,n\in N$$

and meet (1) Non-dimensional; (2) Scaling and (3) Polarization three conditions, thus, this sequence is called comparable.

The four axioms of grey relational measurement

When the space is formed by meeting factor space and comparability, the space is called grey relational space and is demonstrated by { $P(X)\,;\Gamma$}, in which { $P(X)$ } is the theme and Γ is the measurement tool. { $P(X)\,;\Gamma$} have normality; duality Symmetric; wholeness and closeness four axioms.

According to the above descriptions, if a function $\gamma(x_i,\ x_j)\in\Gamma$ can be found to meet all of the above four axioms, $\gamma(x_i, x_j)$ is considered as a grey relational grade.

In grey relational space $\{P(X);\Gamma\}$, exist the sequences $x_i(x_i(1),\ x_i(2),\cdot,\cdot\cdot,\ x_i(k))\in X$.

where $i=0,\ 1,\ 2,\cdots,m$, $k=1,\ 2,\ 3,\cdots,n\in N$ and

$$\begin{aligned} x_0 &= (x_0(1)\,,\ x_0(2)\,,\cdots,\ x_0(k)) \\ x_1 &= (x_1(1)\,,\ x_1(2)\,,\cdots,\ x_1(k)) \\ x_2 &= (x_2(1)\,,\ x_2(2)\,,\cdots,\ x_2(k)) \\ \vdots &= \vdots \\ x_m &= (x_m(1)\,,\ x_m(2)\,,\cdots,\ x_m(k)) \end{aligned} \tag{9}$$

In grey relational grade, if we take $x_0(k)$ as the reference sequence, and the others sequences are inspected sequences, then, it called "localization grey relational grade", if each sequence $x_i(k)$ can be the reference sequence, then, it called "globalization grey relational grade". In our research, we focus on Nagai's grey relational grade [13].

Localization grey relational grade

$$\Gamma_{0i} = \Gamma(x_0(k), x_i(k)) = \frac{\overline{\Delta}_{\max.} - \overline{\Delta}_{0i}}{\overline{\Delta}_{\max.} - \overline{\Delta}_{\min.}} \tag{10}$$

in which $\overline{\Delta}_{0i} = \|x_{0i}\|_2 = \left(\sum_{k=1}^{n}[\Delta_{0i}(k)]^2\right)^{\frac{1}{2}}$

where

$$i=1,\ 2,\ 3,\ \cdots,\ m,\quad k=1,\ 2\ ,\ 3,\cdots,\ n\ ,\ j\in I$$

1) x_0 : Reference sequence, x_i : Inspected sequences

2) $\Delta_{oi}(k) = \| x_0(k) - x_i(k) \|$:

The difference between x_0 and x_i norm.

3) $\Delta_{\min.} = \underset{j\in i}{\overset{\min.\min.}{\forall}}\ \forall k\, \| x_0(k) - x_j(k) \|$

4) $\Delta_{\max.} = \underset{j\in i}{\overset{\max.\max.}{\forall}}\ \forall k\, \| x_0(k) - x_j(k) \|$

Globalization grey relational grade: In the definition of globalization grey relational grade, each sequence can be the reference sequence. In the paper, we still use Nagai's grey relational grade as our mathematics model.

$$\Gamma_{ij} = \Gamma(x_i, x_j) = 1 - \frac{\overline{\Delta}_{ij}}{\Delta_{\max.}} \ , \quad \overline{\Delta}_{ij} = \left(\sum_{k=1}^{n} \left[\Delta_{ij}(k) \right]^2 \right)^{\frac{1}{2}} \quad (11)$$

When the results are found, we can use the eigenvector method to rank the sequence, and then chose the optimal one. The whole steps are illustrated below.

Constructing the relative weighting matrix $[R]_{m\times m}$, which is called “grey relational matrix”.

$$R_{m\times m} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \cdots & \Gamma_{1m} \\ \Gamma_{21} & \Gamma_{22} & \cdots & \Gamma_{2m} \\ \vdots & \vdots & \ddots & \Gamma_{11} \\ \Gamma_{m1} & \Gamma_{m2} & \cdots & \Gamma_{mm} \end{bmatrix} \quad (12)$$

Finding the eigenvalue for the relative weighting matrix $AR = \lambda R$

Using eigenvector method to find the weighting for each target $P^{-1}AP = diag\{\lambda_1, \lambda_{2,} \lambda_{3, ...}, \lambda_n\}$

The maximum $\lambda_{\max}$ corresponding eigenvector are the weighting value for whole sequences.

III. Real Example

A. Subject of Analysis

TABLE I: THE RESULT OF 1ST TEST

No	Mandarin	English	Mathematics	Social	Nature	Total
1	89	96	87	90	82	444
2	86	94	92	79	75	426
3	86	94	97	90	96	463
4	59	39	74	83	70	325
5	72	17	24	77	45	235
6	87	64	86	83	73	393
7	94	100	96	99	96	485
8	86	70	97	93	84	430
9	83	75	59	65	75	357
10	97	96	88	88	76	445
11	97	99	96	96	100	488
13	78	86	92	67	84	407
14	81	50	78	56	58	323
15	84	96	81	94	86	441
16	98	96	93	95	94	476
17	92	88	97	75	80	432
18	94	94	88	84	78	438
19	93	62	89	89	82	415
20	50	36	45	33	44	208
21	97	100	93	96	80	466
22	97	96	95	98	97	483
23	99	97	98	93	94	481
24	82	61	84	67	61	355
25	85	64	64	60	89	362
26	72	55	69	65	60	321
27	97	97	99	98	95	486
28	91	86	100	91	91	459
29	85	67	81	89	83	405
30	65	18	86	77	64	310

For this paper, Changhua County elementary fifth grade students in a class of 29 students are used for the study because the same instructor makes the research more objective. If the study was targeted towards all fifth grade students, the results would lose accuracy due to different teachers. And in the middle of a semester, a total of three exams are taken. The results of the three exams are as follows from Table I to Table III [14].

B. Calculation Results

TABLE II: THE RESULT OF 2ND TEST

No	Mandarin	English	Mathematics	Social	Nature	Total
1	91	81	77	82	62	393
2	85	91	83	84	96	439
3	85	90	83	80	77	415
4	54	20	75	62	59	270
5	61	16	16	71	65	229
6	76	63	59	83	82	363
7	92	96	95	96	92	471
8	86	84	84	85	82	421
9	90	72	56	86	77	381
10	98	96	90	82	93	459
11	95	98	93	93	100	479
13	76	77	83	80	88	404
14	77	33	32	57	64	263
15	88	95	79	81	89	432
16	95	97	91	93	92	468
17	93	73	89	81	76	412
18	93	83	85	91	92	444
19	87	67	87	90	93	424
20	64	16	31	53	63	227
21	91	96	78	92	89	446
22	94	98	96	98	100	486
23	95	96	89	90	100	470
24	87	31	51	61	70	300
25	86	34	69	72	68	329
26	82	56	37	68	66	309
27	92	92	92	94	98	468
28	95	82	80	90	95	442
29	85	55	80	83	91	394
30	71	27	67	79	88	332

TABLE III: THE RESULT OF 3RD TEST

No	Mandarin	English	Mathematics	Social	Nature	Total
1	91	84	94	80	89	438
2	97	93	87	93	95	465
3	89	93	94	90	95	461
4	58	27	88	76	78	327
5	59	10	24	71	67	231
6	91	53	75	87	84	390
7	96	100	98	93	94	481
8	96	83	95	84	96	454
9	85	73	80	96	84	418
10	95	98	91	89	91	464
11	97	99	94	96	96	482
13	83	93	91	79	91	437
14	85	42	56	64	76	323
15	90	97	93	96	87	463
16	99	96	93	92	94	474
17	89	89	83	88	85	434
18	95	76	88	95	91	445
19	87	71	93	86	89	426
20	70	18	28	60	82	258
21	95	97	93	88	91	464
22	99	92	98	98	96	483
23	99	98	98	93	88	476
24	93	48	61	77	75	354
25	86	44	81	76	79	366
26	81	51	31	78	83	324
27	100	96	100	98	100	494
28	96	89	94	94	95	468
29	92	52	90	91	93	418
30	86	14	80	84	95	359

By utilizing the model presented in this paper, the comparison of points of 29 students is used as a case. The working method is as follows:

Listing the comparison of points of ten teacher's evaluations (from Table I to Table III).

Find the weighting of each teacher

- Input the original data
- Transfer original data into normalization data
- Through the grey relational grade to find the weighting of each factor
- Get the weighting decision matrix
- Calculate the whole positive ideal distance and whole negative ideal distance
- Calculate the weighting of each item
- Calculate the weighting of each item

TABLE IV: THE VALUES OF POSITIVE IDEAL DISTANCE AND NEGATIVE IDEAL DISTANCE: 1ST TEST

Item / No	No:1	No:2	No:3	No:4	No:5
S_i^+	0.0003	0.0004	0.0003	0.0003	0.0003
S_i^-	0.0005	0.0005	0.0008	0.0005	0.0005

TABLE V: THE WEIGHTING VALUE OF EACH OF EACH ITEM: 1ST TEST

Item / No	No:1	No:2	No:3	No:4	No:5
C_j	0.6419	0.6670	0.6910	0.6414	0.6380

TABLE VI: THE VALUES OF POSITIVE IDEAL DISTANCE AND NEGATIVE IDEAL DISTANCE: 2ND TEST

Item / No	No:1	No:2	No:3	No:4	No:5
S_i^+	0.0003	0.0006	0.0005	0.0003	0.0003
S_i^-	0.0002	0.0009	0.0009	0.0003	0.0003

TABLE VII: THE WEIGHTING VALUE OF EACH OF EACH ITEM: 2ND TEST

Item / No	No:1	No:2	No:3	No:4	No:5
C_j	0.4555	0.6007	0.6306	0.5576	0.5000

TABLE VIII: THE VALUES OF POSITIVE IDEAL DISTANCE AND NEGATIVE IDEAL DISTANCE: 3RD TEST

Item / No	No:1	No:2	No:3	No:4	No:5
S_i^+	0.0001	0.0006	0.0003	0.0003	0.0001
S_i^-	0.0005	0.0009	0.0008	0.0003	0.0002

TABLE IX: THE WEIGHTING VALUE OF EACH OF EACH ITEM: 3RD TEST

Item / No	No:1	No:2	No:3	No:4	No:5
C_j	0.7932	0.5969	0.6965	0.4853	0.6866

TABLE X: THE MEAN OF WEIGHTING

Item / NO	No:1	No:2	No:3	No:4	No:5
C_j	0.6302	0.6215	0.6727	0.5614	0.6082

C. Development of Toolbox

From the calculation steps in the mentioned above, if we following the calculation steps of Grey Relational Grade-TOPSIS model to get the results, which show from Table IV to Table VII are very boring and easy to make mistake. Hence, in the paper, the toolbox is developed to help the huge data calculation and analysis, the main calculation of toolbox are shown form are shown from Fig. 1 to Fig. 3 [15].

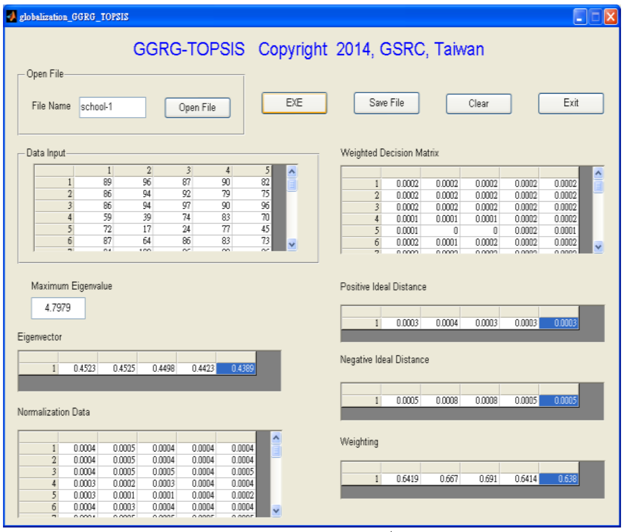

Fig. 1. The results of 1st test.

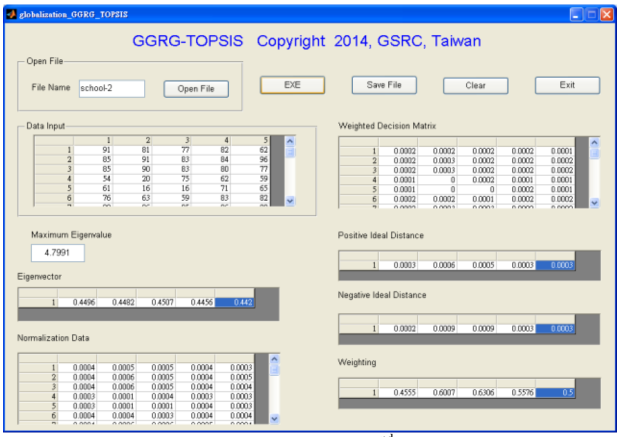

Fig. 2. The results of 2nd test.

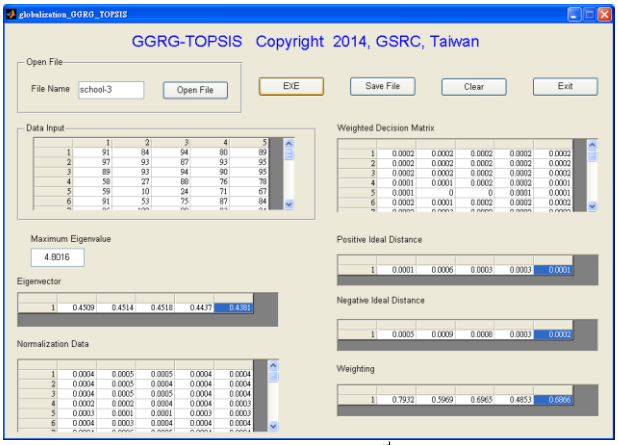

Fig. 3. The results of 3rd test.

IV. CONCLUSION

Due to weighting is very subjective, in the previous researches, they all want to reduce the subjective component to achieve objective. There are many publications so far. The paper first used the grey relational grade to calculate objective weighting and integrated with TOPSIS. Also, the self-developed Matlab toolbox is used to analyze the data, and it enables the objectiveness of the analyzed system. Through the examples of calculation and verification, we found that although the weighting obtained by the new approach seemed to be the same, it can still prove the weighting is achieved by objective calculation. To sum up, for the weighting analysis, the paper only used one of mathematics method in the grey system theory. In the future, it is suggested to increase the other soft-computing calculation method to develop more rapid mathematics software toolbox, and enhance the level of education of the work or suggest in others application.

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