Analysis Learning Obstacle on Quadratic Function Topic

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***Abstract*—Learning quadratic functions is to combine aspects of geometry and algebra in which this topic becomes crucial for students to understand fully. Unfortunately, they often experience barriers in studying quadratic functions. This article aims to elaborate on what learning obstacles the students experience in studying quadratic functions. Each student has different characteristics, so it leads to different difficulties as well. By recognizing the learning obstacles, teachers can help them overcome those barriers, so that the concept of understanding on the topic of quadratic function is not gained partially.**

***Index Terms*—Learning obstacle, quadratic function, student understanding.**

I. Introduction

Mathematics is a discipline taught from basic education to higher education level consisting of several topics. Mathematical topics given to students in elementary to secondary education level include: numbers and operations, algebra, geometry, measurement, and data analysis and measurement [1]. One of the topics where they still experience obstacles is on function, especially quadratic function. This topic is essentially a merger between algebra and geometry topics. In this case, the two topics are those becoming the students' barriers in understanding mathematics. Thus, to be able to understand the topic, the teachers must be able to design a learning that is able to bridge their understanding of algebra and geometry.

In the process of learning mathematics, it basically relates to three scopes: teachers, students, and materials [2]. These things must go hand in hand for the success of the learning process. In addition, the learning success is also determined by how the teacher designs the learning process. Learning design is the teacher's main responsibility through designing teaching and learning activities [3]. In its process, the teacher must involve a complex while unique set of processes. It starts from conducting curriculum analysis to determine what topics and lesson themes will be delivered [4]. In this case, the teacher will make a learning trajectory about the materials presented to the students. Learning trajectory helps teachers understand the students' level of knowledge as the key to present what they need [5] It is pinpointed that teachers should understand the materials to be delivered.

Based on the results from Istiqomah's study [6], there are some epistemological barriers related to function topic, namely: 1) epistemological obstacles related to the existing conceptual image about the definition of function, 2) epistemological obstacles related to the contextual information variation available on questions, 3) epistemological obstacles associated with the students' ability to translate the existing information into functional notation, and 4) epistemological obstacles on the connection of function concepts to other mathematical concepts especially in the concept of numbers, equations and algebraic operations.

Learning obstacles experienced by the students lead to the level of mathematical ability they have. One of the efforts in improving their mathematical ability is through constructive learning. The National Council of Teachers of Mathematics states that students must study mathematics with understanding, actively building new knowledge based on prior experience and knowledge [1]. It is the teacher's task to help and scaffold the students in building their own understanding of the materials, especially on the topic of quadratic function. The basic knowledge that they must possess in understanding such material is to understand how to interpret the signs used in algebraic operations, variables, and algebraic expressions and equations to relate the understanding of algebra to the understanding of geometry and vice versa.

Based on those explanations, the formulation of problem is what learning obstacles are experienced by the students regarding quadratic function lesson?

II. Theory

Learning is basically a process of building self-reliance through the actualization of authority and learning rights of the students. In the process, they are entitled to learn (think, argue, ask) or otherwise, and have the rights to create learning situations. In practice, they may naturally experience a situation called learning obstacles. These can be caused by several factors. There are three factors that cause the emergence of learning obstacles: ontogeny (mental readiness), didactic (teacher's teaching), and epistemology (students' knowledge limitation on contextual application). In identifying the students' obstacles while studying a material, the most closely related factor is epistemological one [7].

According to Doroux epistemological obstacle is essentially a person's knowledge that is limited to a particular context. If the person is faced with a different context, his or her knowledge becomes unusable or is having trouble using it [2]. For example, a person who initially learns the concept of equation is only confronted with a conventional model with algebraic operations on the left and its results on the right, the concept image, thus, represented in the student's mind is that an equation must always be that way. When he is faced with a different problem concerning on algebraic operations on the right and its results on the left, or algebraic operations on

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both sides, then unexpected obstacles will likely arise.

A. The Bruner Theory (Three Modes of Representation)

Learning mathematics will be more successful if the teaching process is directed to the concepts and structures contained in the subject matter taught. By understanding the concepts and structures covered in the subject being discussed, the learners will understand the material to be mastered. This suggests that matter that has a particular pattern or structure will be more easily understood and remembered by the learners [8].

Three modes of representation in a child's mental development. They are enactive, iconic, and symbolic. In an enactive stage, the child learns to use objects directly or to use concrete objects. The iconic stage states that the child can already manipulate using images of objects. The last stage is the symbolic stage. According to Bruner, at this stage the child manipulates directly by using mathematical symbols and has nothing to do with objects [8].

1) Construction theorem

This theorem states that if the child wants to have the ability to master the concepts, theorems, definitions and so on, the child must be trained to carry out the preparation of his representation. To embed certain ideas or definitions in mind, children must master the concept by trying and doing it themselves. Thus, if the child is active and involved in learning the concept by showing the representation of the concept, then the child will be more understanding. It can be concluded that essentially, in the early stages of conceptual understanding, concrete activities are required that bring the child to the concept.

2) Notation theorem

The notation theorem reveals that in the presentation of concepts, notation plays an important role. The notation used in declaring a particular concept must be matched with no child's mental development. This means to express a formula for example, then the notation must be understandable to the child, not complicated and easily understood.

3) Contrast and diversity theorem

In this theorem it is stated that contrast and diversity are very important in making the concept change understood in depth, it takes many examples, so that children are able to know the characteristics of the concept. The child needs to be given an example that satisfies the formulation of a given theorem. In addition they should also be given examples that do not meet the formulation, nature or theorem, so expect the child does not experience a misunderstanding of the concept being studied.

The concept described by example and not example is one way of contrast. Through this way the child will easily understand the meaning of the characteristics of the given concept. For instance, to describe right triangles, it is necessary to give examples whose images are not always upright with the bevelled side in a tilted position, but it should also be given a picture with sloping sides in a horizontal or longitudinal state. In this way the child is trained in examining whether the triangle given to him is classified as a right triangle or not.

4) Connectivity theorem

This theorem states that in mathematics, there is a close relationship between one and other concept, not only in terms of content, but also in terms of the formulas used. One material may be a prerequisite for others, or a particular concept is required to explain other concepts.

Teachers need to explain how the relationship between something is being explained with another object or formula. Whether the relationship is in the same formula used equally can be used in the field of application or in other matters. Through this way the child will know the importance of the concept being studied and understand how the formula or idea he is currently learning in mathematics. Children need to realize how the relationship, because between a discussion with other mathematical discussions are related.

5) Piaget theory

Based on the results of his research, Piaget suggests that there are four stages of individual cognitive development that develops chronologically based on age, namely: 1) stage Motoric Sensory (age 0-2 years), the stage where experience is obtained from physical actions (limb movements) and sensory (sensory coordination); 2) Pre-Operation stage (age 2-7 years), i.e. preparation stage for organizing concrete operations; 3) the stage of Concrete Operation (age 7-11 years), i.e. the stage of understanding logical operations with the help of concrete objects; 4) Formal Operation stage (11 years and above), i.e. the stage in which the student has been able to do the reasoning by using abstract things.

6) APOS theory

Dubinsky introduces the process of forming new knowledge (especially in mathematics) which is believed to be the result of a series of Action-Process-Object-Schema (APOS) processes [9]. APOS is a constructivist theory of how one learns a mathematical concept. The theory is based on the hypothesis of the essence of mathematical knowledge and how it develops. Objects that have been stored in someone's memory as mathematical knowledge will be processed because of the action to respond to a certain stimulus or problem situation through reflection on the problem and its solution in a social context. The reflection is performed through the construction of actions, processes, and mathematical objects and organizing them in a scheme that can be used in relation to the problem situation at hand.

The terms action, process, object, and schema are essentially a mental construct of a person in an attempt to understand a mathematical idea. According to the theory, when one seeks to understand a mathematical idea the process will start from a mental action to the mathematical idea, and leads to the construction of a scheme of certain mathematical concepts covered by the given problem.

Action is a transformation of mental objects to acquire other mental objects. When facing a problem and trying to relate it to the knowledge already possessed, the individual experiences an action. A person is said to experience an action if the person focuses his mental processes on trying to understand a given concept. Someone who has a deeper understanding of a concept will probably do a better action. On the other hand, it can also focus someone's attention out of the concept given so that the expected action cannot happen.

When an action is repeated, then a reflection of action takes place, it will enter the process phase. In contrast to the action which may occur through the aid of the manipulation of concrete objects, the probe proceeds internally under the control of the individual who does so. If one thinks of the limited mathematical ideas encountered and characterized by the emergence of the ability to explain or reflect on mathematical ideas, then it can be said that a person experiencing a process of a concept covered by the problems it faces. Through a coordination and inter-process linkage, new processes can be constructed from other processes.

A person is said to have constructed the process into an objectivist object, if the individual is reflecting on the operations used in a particular process, becoming aware of the process as a totality, realizing that certain transformations may apply to the process, and being able to carry out the transformation in question. It can be said that the processes performed have been encapsulated into a cognitive object. Individuals can be said to have had an object conception of a mathematical concept when he was able to treat the idea or concept as a cognitive object that included the ability to act on the object and provide a reason or explanation of its properties. In addition, the individual has also been able to perform de-encapsulate of an object into a process as it originated when the properties of the object in question will be used.

The set of actions, processes, objects, and other interconnected schemes in the individual's mind so as to form an interrelated framework, is a scheme of a mathematical material.

B. Zone of Proximal Development (ZPD) Vygotsky

Various stored mental processes can be generated through learning process and can be operated when someone interacts with an adult or collaborates with one another [10]. At the time of solving the problem by self-learning process, individuals can develop the ability that is called actual development. In addition, there is potential development which is a development that occurs as a result of interaction with teachers or other students who have more ability. The distance between actual development and potential development is called zone of proximal development.

According to Vygotsky, the learning process occurs in two stages, the first stage occurs when collaborating with others and in the next stage is done individually in which the internalization process occurs [10]. During the interaction process between teachers and students, there are several capabilities that need to be developed: mutual respect, testing the truth of others' statements, negotiating, and mutually adopting developing opinions.

The exchange of different learning experiences is expected to occur through interaction between students so that the expected mental actions can continue. Mental action is expected to occur well by providing advanced challenges through scaffolding techniques, in addition to using these techniques to direct individual thought processes. This kind of activity can continue until the student has the ability to reflect on the mental actions taken. This can be seen in part from students' ability to discuss the results of mental actions that have been done on a number of related cognitive objects.

III. Discussion

Learning obstacles that arise from quadratic function lesson are concerned in several studies. A study was conducted by Metcalf on three of his students at New England University [11]. He found that only one participant could perform several procedures, indicating the limited relational understanding of the concept. However, no single participant can demonstrate flexibility in representing and communicating the concept of quadratic function.

Then, Leinhardt et al. found that students experienced misconceptions about the function lesson, namely: 1) what function is and otherwise; 2) correspondence within the concept of function; 3) how to generalize properties in linear function; 4) continuous and discrete graphs; 5) various representations of equivalent function; 6) relative reading and interpretation; 7) the concept of variables in function; 8) notation on the graph of the function itself [12].

In addition, Kotsopoulos found that middle-class students experienced obstacles when performing squared factorization. Such difficulties arose when they were asked to repeat the multiplication of facts [13]. He stated that they were confused at unusual questions as they usually do, for example: $x^2+3x+1=x+4$.

Rahmawati, Suparta & Suweken found some obstacles when students were exposed to deal with issues about quadratic function, namely: 1) they did not understand the purpose of the problems; 2) they felt confused in using the "formula" because they are accustomed to work by using formulas; 3) they tend to memorize the way of problem solving; 4) they are not used to using elaboration; and 5) some of them solved the problems in groups, not independently [14].

The same obstacles were also found by Ellis & Grinstead regarding quadratic function: 1) the relationship between algebra along with how to solve problems with tables and also graphical representations; 2) graphical display as the whole object; 3) the struggle to correctly interpret the role of the parameters; and 4) the tendency to make a false generalization regarding linear function [15]. They found those difficulties associated with algebra with graphical representation in which two of the three students they interviewed explained a rule with parameter: a in $y=ax^2+bx+c$ as a slope of quadratic function. This is not true since the slope is not part of quadratic function, but is part of linear function.

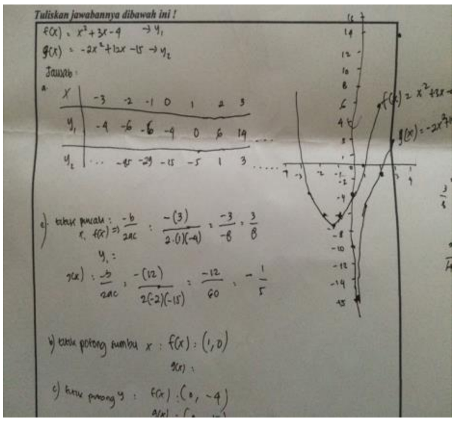

Fig. 1. The result of observation with 4 students.

Based on the observation conducted on four high school students who have studied the material of quadratic function as seen on figure 1, the learning obstacle obtained are: 1) the observed students have difficulty in determining the abscissa value and ordinate of a quadratic function; 2) the observed students have no geometric understanding to determine the symmetrical axis of a graph of quadratic functions; 3) the observed students mostly only memorizes the formula to determine the symmetric axis; 4) the observed students have difficulty to distinguish what is the minimum and maximum points with a symmetrical axis; 5) the observed students have not been able to make a function graph properly.

It can summarized that some common obstacles experienced by the students include: 1) students' difficulties in interpreting the information contained in the function graph; 2) the relationship between quadratic function and quadratic equations; 3) the similarities between quadratic function and linear function; 4) the inability to re-shape quadratic function with parameter 0; and 5) students' difficulties in solving quadratic function factorization.

There are several solutions offered to overcome the learning obstacles regarding quadratic function. Rahmawati, Suparta & Suweken disclosed that they can be overcomed by visual-based scaffolding [14]. Parent revealed one solution that can be used to reduce the learning obstacles through re-formulating the existing quadratic rule to be more easily recognized by the students or commonly known representations [16]. Schiro said that teaching involves three basic operations: careful diagnosis and observations of students and individual needs and interests; making sure the physical, social, emotional, and intellectual environment in which the students can learn; and facilitating students growth by intervening between them and their environment [17].

IV. Conclusion

In addition, another way that can help reduce students' learning obstacles regarding quadratic function lesson is to create a learning design in accordance with their characteristics and materials' characteristics that will be taught, so that their understanding of quadratic function is not gained partially.

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