Problem-Based Learning: A Tool for the Teaching of Definite Integral and the Calculation of Areas

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***Abstract*—One problem that afflicts teachers, particularly those who teach Differential and Integral Calculus, is the difficulty of students in mathematics content and the consequent lack of understanding of what is presented in this discipline. Numerous studies have been carried out with the aim of creating and improving new teaching-learning methodologies that make students active elements of the learning process. This work proposes the application of the Problem Based Learning methodology in the definite Integral topic of the Calculus discipline, in order to guide the teacher and contribute to the student having contact with the discipline in problems that represent situations closer to their reality. In this proposal the student will also make use of technologies that, although they are known, are often underutilized by them.**

***Index Terms*—Area calculation, definite integral, Problem-Based Learning, real problem.**

I. Introduction

The initial years of Engineering courses in the country include in their curriculum the so-called "basic cycle" that accommodates some important disciplines, among which "Differential and Integral Calculus" stands out. With respect to these courses, the Differential and Integral Calculus disciplines play a fundamental role in the training of these future professionals, since it enables the mathematical modeling of real problems of several areas of an engineer's performance, and allows a more sophisticated study on the problem in question. However, what is noticeable is that most of the students present difficulties to apply the concepts learned in the disciplines of calculation in situations that represent real problems. These difficulties are due to several causes, but in terms of aspects of the subject itself, we can highlight the extensive content to be fulfilled, the student's lack of encouragement to understand the concepts and fundamental principles of the contents addressed in the classes, the lack of time for the teacher working with different devices, among others, reflecting a high rate of reproofs.

The high number of failed students is the subject of long discussions between teachers in order to improve the performance of these students and consequently make them able to understand the importance of the tools and concepts learned, as well as to relate the calculation to other disciplines.

For decades, many researchers have been focusing on the creation and improvement of new teaching methodologies with the aim of making students less passive during classes and more engaged in the learning process. Recent works have evaluated the level of motivation and performance of Calculus students when subjected to Problem-Based Learning (PBL) and to the traditional Teaching Method [1], [2]. The results showed that motivation, relevance, confidence, and satisfaction attention of students is greater using the PBL methodology compared to the traditional teaching method. According to the students, the PBL approach enables them to take an active role in class, increasing success and gaining self-confidence [1]. The use of new tools that help students to interpret abstract concepts are also elements that propel the learning process. In [3] the TINspire graphing calculator was used in a mixed skills classroom at a public high school. The results indicated that TINspire helped reinforce students' understanding of the concepts of mathematics as well as stimulate logical thinking. Other teaching learning methodologies can also be used in Calculus classes, as shown [4]. In this work the so-called Peer Instruction is used in the Calculus course for engineering students in order to replace traditional classes. In this case, the results showed that students liked to be more active and motivated with the Peer Instruction, but also observed a problem regarding the difficulty in reading the textbook in advance.

Returning to the issue of Differential and Integral Calculus teaching in the engineering course, it is known that the time required for the concepts and theorems involved to be properly dealt with, in general, encompasses the total workload of the discipline, and it is not possible to apply projects or new methodologies. In this section, this article proposes the use of the PBL methodology in a topic of Differential and Integral Calculus with the purpose of bringing to the student the issue of interdisciplinarity, besides integrating theory and practice in problems that involve real situations and the use of technologies that can give support the solution of the problem involved.

II. Problem-Based Learning

Currently, several teaching-learning methods have been applied in the most several areas in order to bring the student not only knowledge, but also make it capable of to solve problems and take initiatives. In this work is approached the methodology called **Problem-Based Learning (PBL)**. PBL is a collaborative, constructivist and contextualized teaching-learning method that makes use of problems (real or simulated) that are directly related to the future performance of students as professionals and citizens [5]. Thus, PBL acts not only in the needs of students, since they stimulate them to

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research, to be critical and to make decisions, but also attends to what teachers and society expect from future professionals, as it allows the student to make connections between the disciplines and knowledge [6]. The PBL proposal meets the current needs of society, where we live in a globalized scenario where changes in the professional scope occur fast and daily, which requires more and more professionals able to search for solutions in a practical and intelligent way.

In order for the PBL to be applied in a way that adequately serves the target audience, regardless of school level, course or discipline, it is important to follow a basic organization governed basically by five steps [7]:

1) Identification of the problem: the teacher defines the problem that will be studied, prepares and systematizes the materials necessary for the investigation.
2) Discussion of the problem and hypothesis: students elaborate the questions about the context of which they have prior knowledge and that will deepen. The group should then initiate research planning for problem resolution.
3) Application of knowledge to the problem: students should use prior knowledge, materials made available by the teacher, technological support or any information or tool that makes it possible to solve the problem.
4) Presentation of the solutions of the group: the group should synthesize the discussions and reflections, systematize the solution found for the problem and prepare a presentation for the teacher, being in the form of report, oral presentation, written presentation, etc.
5) Self-evaluation of the learning process they have done.

III. PBL Applied to the Teaching of Calculus in Engineering

There is a concern in the engineering courses that the classes not only can give solid support of technical-scientific knowledge, but also that they can arouse some professional attributes such as the ability to work in team, ethics, professional and social responsibility. As for the discipline of Calculus, in particular, the theoretical classes themselves are poor in the matter of working some skills and attitudes important for the future professional. With the proposal to work the calculation concepts, skills and attitudes, and still without taking much of the precious time of classes, in the proposal that follows will be considered the PBL methodology when introducing a problem that integrates the topic of the calculation content, **definite integral**, with a real scenario, integrating interdisciplinary knowledge.

For the development of the activity the teacher should separate the students into groups and then expose the problem to be worked:

Problem: "**Calculate the area of a region in country that has social or ecological importance**".

Once the problem is released, the following steps should be followed:

1) Students should search for regions, for example, that have a large accumulation of industries, deforested regions, among others, and write the importance of calculating the area of this region;
2) The group should delimit the region using computational tools such as Google Earth, Google Maps, etc.;
3) After delimiting the region, the group should discuss and raise hypotheses on how the area can be calculated;
4) The group should use some software, for example GeoGebra, to find the curves that best represent the boundaries of the region and the functions that define them;
5) The calculation of the area should be established by the group, using the definite integral's concept besides previous knowledges to assist them in solving the problem.

A brief review on the concept of definite integrals will be carried out and finally the results of a paper presented by a group of students of the basic engineering cycle will be presented, with the purpose of illustrating the proposal of this article.

A. The Definite Integral and the Areas Calculus

From ancient times the concern with the problem of determining the area of flat figures had afflicted mathematicians. As the area of some flat regions were well known, then the procedure leading to a satisfactory result of the area of regions that did not have a proper formulation was the method of exhaustion. Such a method was to approximate the plane figure by means of polygons whose areas could be calculated by the methods of elementary geometry.

To demonstrate the method of exhaustion, consider the problem of determining the area of a flat region *S*, shown in Fig. 1, delimited by the graph of a nonnegative continuous function *f*, by the *x*-axis and by the straights $x = a$ and $x = b$ [8].

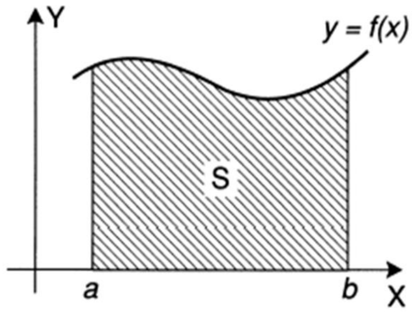

Fig. 1. Flat region S [8].

Consider the partition *P* of the interval [*a*, *b*], defined by dividing the interval into *n* subintervals, such that $a = x_0 < x_1 < ... <x_{i-1} < x_i < ... < x_n = b$. Let the length of the interval $[x_{i-1}, x_i]$ given by $\Delta x_i = x_i - x_{i-1}$. For each *i*, $i = 1, ..., n$, we construct a base rectangle Δx_i and height defined by any point c_i that belongs to the interval $[x_{i-1}, x_i]$. Fig. 2 shows some rectangles formed by the partition *P* of the region *S*.

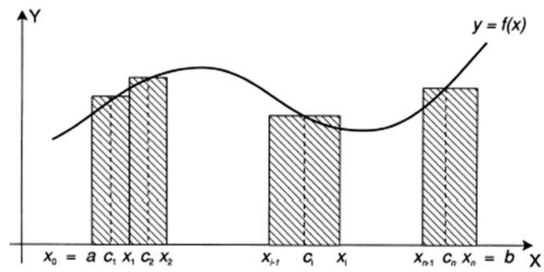

Fig. 2. Rectangles formed by the partition *P* of the interval that defines the region *S* [8].

The area of region S can be approximated by the sum of n rectangles, represented by S_n, and given by [8]

$$S_n = f(c_1)\,\Delta x_1 + f(c_2)\,\Delta x_2 + \ldots + f(c_n)\,\Delta x_n, \quad (1)$$

and can be represented by

$$S_n = \sum_{i=1}^{n} f(c_i)\,\Delta x_i. \quad (2)$$

Eq. 2 is called the **Riemann Sum** of the function $f(x)$. Fig. 3 shows the case of a partition for $n = 8$. Note that the area obtained by the sum of the 8 rectangles, S_8, is an approximation of the area of the region S. As the value of n grows, the values of x_i become too small, making the sum of the rectangular areas close enough that we intuitively understand as the area of S.

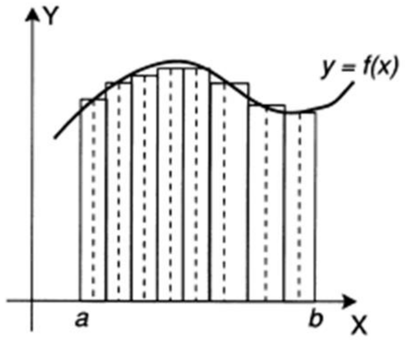

Fig. 3. Rectangles formed by the partition P of the interval for $n = 8$ [8].

From these concepts, the area of any limited region can be obtained from the following definition [8]:

Definition 3.1 Let $y = f(x)$ be a continuous nonnegative function on $[a, b]$. The area under the curve $y = f(x)$, from a to b, is defined by

$$S = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i. \quad (3)$$

The definite integral is a terminology that originated from the mathematical formalization of area problems and physical problems and is directly related to the limit of Definition 3.1. The following is the definition that relates the defined integral to the limit presented in Eq. 3.

Definition 3.2: Let f be a function defined in the interval $[a, b]$ and let P be any partition of this interval. The definite integral of f from a to b is given by:

$$\int_a^b f(x)\,dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i. \quad (4)$$

since that exists the limit of the 2nd member. In this case f is an integrable function in $[a, b]$.

The purpose of this section is to present the basic concept involved in the problem that is introduced to students, as well as to illustrate the relationship between geometry and mathematical development that characterizes the definite integral and the area calculation. The integral's properties and the fundamental theorem of the calculation, which allows to relate derivation and integration operations, can be found in more detail in Calculus books [8], [9].

B. Application: Calculation of the Flooded Area of the Imboassica Lagoon in Macaé - RJ

In this section will be presented briefly the project developed, in the discipline of Calculus II, by a group of 4 students who study Engineering at a Public University of the State of São Paulo in the second half of 2018.

The developed project presents an application of the Definite Integral whose objective is to calculate the area flooded by the overflow of the Imboassica Lagoon, located in Macaé - RJ. This lagoon has great tourist importance for the region, being flooding problematic for the local residents and for the activities performed in it. The format of the flooded region can be approximated by the combination of certain functions, and the area calculation is obtained through defined integrals. GeoGebra, Google Maps and Google Earth software were used to facilitate the visualization of the region and the calculations involved.

1) A little about the history of the Imboassica Lagoon

Every river or body of water (oceans, seas, lagoon, etc.) has an area around it that usually floods at certain times of the year or when there is a very large precipitation index increasing the flow and causing an overflow [10]. Such floods are natural phenomena that occur in all bodies of water. The setback is that with the construction of real estate on the banks of rivers, which do not respect this natural limit of overflow, such a natural phenomenon can become dangerous and directly affect the health and mobility of the locals. Some factors that contribute to the worsening of these floods can be highlighted:

- The waterproofing of the soil by the asphalt and concrete, reducing the flow of water that would be drained,
- The discard of garbage by the population in inappropriate places, in which there is no garbage collection, causing the movement of these wastes,
- Unplanned or irregular constructions,
- Human interference with the environment through deforestation, burning or even the elimination of native vegetation, among others.

The state of Rio de Janeiro is characterized by the formation of coastal lagoons, environments formed mainly by historical of sea level, coastal hydrodynamics and the amplitude of tides [11]. The Imboassica Lagoon, located in the municipality of Macaé in the State of Rio de Janeiro is one of the cases where the local population suffers from the floods. The urban occupation in the Imboassica Lagoon began in the mid-1960s, with a large increase in oil exploration in the Campos Basin and the population growth of the city of Macaé [12].

With the reduction of the riparian forest and the disordered increase of the population around the lagoon, as shown in Fig. 4 and Fig. 5, flooding in some neighborhoods is frequent. To avoid this kind of problem, bar opening is necessary. The term "bar opening" is due to the breaking of the sandy cord that separates the coastal lagoon from the sea [13], which can occur naturally, by raising the water level or artificially by man. In the case of the Imboassica Lagoon this opening does not happen naturally, obeying the cycle of nature, but rather through human action (often irregularly), which is the subject of discussion by environmentalists and institutes responsible for the environment [10].

Fig. 4. Image taken from Imboassica Lagoon, in the municipality of Maca é– RJ.

Fig. 5. Image obtained by satellite (adapted from Google maps).

Motivated by the frequent flooding problem that affects the population dynamics in neighborhoods close to the occurrence of the phenomenon, it was decided to study the flooded area due to the overflow of Imboassica Lagoon, located in the city of Maca é - RJ.

2) The mathematical development

Considering the floods that already occurred in the north of the Imboassica Lagoon, a certain region was assumed in the Mirante of Lagoon neighborhood, shown in Fig. 6, and the objective was to find the approximate value of the highlighted area.

Fig. 6. Red highlighted region delimits the flooded area of Mirante of Lagoon neighborhood (Adapted from Google Earth).

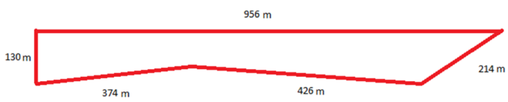

Fig. 7. The dimensions of the region to be calculated the area.

Using computational tools such as Google Earth, it is possible to determine the approximate distances of the lines defining the region boundary in Fig. 6. The shape of the flat region is shown in Fig. 7.

GeoGebra software allows the inclusion of images to be referenced in a Cartesian system. Fig. 8 shows the main points (A, B, C, D, E) in the image delimiting the region of interest.

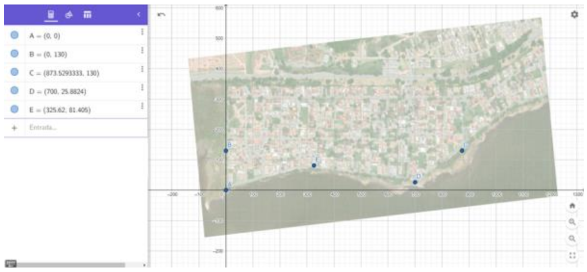

Fig. 8. Image of Imboassica Lagoon referenced in Cartesian system of Geogebra software.

From Fig. 8 it was possible to define the functions, in the linear case, that represent the boundary of the flooded region. The functions and curves representing them are shown in Fig. 9.

Fig. 9. Functions that delimit the flooded region and their respective graphs.

The polygon shown in Fig. 9 was divided into three regions for the calculation of the total area *A*. The regions are shown in Table I.

TABLE I: FUNCTIONS THAT DEFINE AREA CALCULATION PER REGION

Region	Interval	$y = S(x)$	Area
S_1	$0 \leq x \leq 325.62$	$y = i(x) - f(x)$	$S_1 = 130 - x/4$
S_2	$325.62 \leq x \leq 700$	$y = i(x) - g(x)$	$S_2 = 0.148x + 0.4$
S_3	$700 \leq x \leq 873{,}53$	$y = i(x) - h(x)$	$S_3 = -0.6x + 524.12$

TABLE II: PRESENTATION OF THE NOTATION OF THE DEFINITE INTEGRAL PER REGION, PROCEDURE AND RESULT OF THE AREA

Definite Integral	Calculation of the definite integral	Result of area (m^2)
$S_1 = \int_0^{352.62} \left(130 - x/4\right) dx$	$S_1 = \left[130x - 0.25x^2\right]_0^{352.62}$	S_1 = 30,297.99
$S_2 = \int_{352.62}^{700} (0.148x + 0.4)\,dx$	$S_2 = \left[0.074x^2 + 0.4x\right]_{32.62}^{700}$	S_2 = 27,197.73
$S_3 = \int_{700}^{873.53} (-0.6x + 524.12)\,dx$	$S_3 = \left[-0.3x^2 + 524.12x\right]_{700}^{873.53}$	S_3 = 9.033,73

Having defined the functions that represent the area for each region (S_1, S_2, S_3), fixed at intervals on the horizontal axis *Ox*, the definite integral applies. The procedure for calculating the area is presented in Table II.

The neighborhood lagoon viewpoint total area is approximately:

$$S = S_1 + S_2 + S_3$$

$$S = 66{,}529.86\,\text{m}^2$$

Having information on the size of the regions that suffer from the flood problem is important, as it enables the monitoring and management of the risk areas, making possible the city's action plans.

It is important to emphasize that, even though the region considered to be relatively simple, since it is basically formed by straight segments, the computational tools used provide the necessary support for whatever the format of this region, as can be found in other works [14].

IV. Final Considerations

The main objective of this work was to present to the teacher a proposal of application of the ABP methodology in the content of Definite Integrals that is approached in the classes of Differential and Integral Calculus. To illustrate the procedure was shown the result of a work developed by a group of students of Calculus of Engineering course. It was observed the interaction among students, the search for learning to use new tools, the ability to associate interdisciplinary issues, and the possibility to perceive in a more practical way the importance and application of the concepts seen in class. The application was presented enables the student to make use of computational tools such as Google Earth, Google Maps, GeoGebra, among others, to assist them in the development of the work, since when dealing with real situations it is not always easy to analyze the geometry and calculations involved.

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