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Applying Dynamic Geometry Environment Software as a Visualization Tool for Teaching Planimetry Construction Tasks

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***Abstract*—The present study addresses a notable gap in the current pedagogical research by rigorously examining the impact of Augmented Reality (AR) on mathematics education, specifically focusing on two-dimensional (2D) plane geometry construction tasks. Our primary aim is to discern and articulate the unique contributions of AR as a pedagogical tool in this domain. Employing a quasi-experimental design, we engaged with 141 high school students, dividing them into two distinct groups: one receiving instruction through AR tools and the other via traditional teaching methods. Performance metrics were derived from pre- and post-construction tests as well as an attitudinal survey gauging students' perceptions of AR. Our findings reveal that the AR-facilitated group showcased a marked improvement in post-test scores and exhibited a more positive attitude towards AR technology compared to the traditionally taught cohort. This point towards AR's potential to significantly elevate comprehension and engagement levels in 2D geometry construction. The study further highlights the transformative potential of AR in bolstering students' enthusiasm and curiosity in mathematical endeavors. The results presented herein emphasize the pressing need for extended research to gauge AR's broader and sustained effects in mathematics education and its applicability across various mathematical disciplines.**

***Index Terms*—Construction problem, dynamic learning environment, education, plane geometry, visualization**

I. INTRODUCTION

In recent years, information technology's rapid evolution has revolutionized pedagogical approaches across various academic fields, prominently within geometry [1]. The advent of computers has not only modernized our educational strategies but has also sparked the transformation of conventional teaching methodologies [2]. Yet, amidst these advancements, a notable gap persists: the under-explored potential of using these technologies for geometric concepts, especially planimetry construction tasks.

The relevance of this research stems from its intent to bridge this knowledge chasm. As the academic world grapples with integrating digital innovations into geometry teaching, understanding the applicability of specific tools becomes paramount [3]. This study's findings could redefine teaching norms and set new benchmarks, making it indispensable for educators and stakeholders.

The rapid digital transformation in education presents both challenges and opportunities [4]. For our diverse readership, which includes educators and curriculum designers, understanding the nuances of this shift is vital. Grasping how to adeptly utilize technology can significantly enhance the teaching of geometric concepts [5]. By integrating innovative tools, these complex ideas can be rendered more accessible, fostering a more interactive and immersive learning environment.

Existing literature has extensively acknowledged the digital evolution in geometry education, yet a significant lacuna persists [6]. Many of these studies have not exhaustively explored the specific applicability and efficacy of dynamic geometry environment software as a pivotal visualization instrument. Our research seeks to address this oversight, contributing a nuanced understanding to this specialized domain.

Responding to the pressing need for innovative teaching methods, especially those aligning with Federal State Educational Standards, this research foregrounds the principle of visibility [7]. By leveraging dynamic geometry environment software, educators can potentially amplify students' visual thinking capacity, offering a more interactive and immersive learning experience [8]. The nuanced understanding of transforming visual data into tangible cognitive knowledge [9] can set the stage for a groundbreaking pedagogical approach.

At the heart of our investigation lies the salient question: "How can dynamic geometry environment software reshape the pedagogy and understanding of plane geometry construction tasks?" Engaging with this question aims to bridge the prevailing research chasm, providing a comprehensive insight. Furthermore, the answers derived promise to present an informed framework, emphasizing the judicious integration of technology in geometric instruction.

In an era where education's digital facet dominates, characterized by virtual interfaces and diverse online platforms, this study stands distinct [10]. While these platforms democratize and simplify access to education, integrating them purposefully in teaching is the challenge. Our research pinpoints the transformative power of dynamic geometry environment software, positioning it as a crucial element in modern geometry education. This emphasizes our unique contribution to the literature, offering actionable insights for a digitally-integrated pedagogical future.

II. RELATED WORKS

A. Dynamic Geometry as a Visualization Tool

Today, the problem of the formation of spatial thinking of

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schoolchildren is not new to the methodology of teaching mathematics, its relevance has been discussed and written for quite a long time. One particular example is the propaedeutic course “Visual geometry 5th-6th grade” [11, 12]. However, this work mainly features the traditional approach operating with static interpretations of geometric concept, while the use of information technologies in the classroom makes it possible to reevaluate conventional approaches to the study of numerous geometry problems. Here the question of the use of dynamic interpretation of geometric concepts using various software tools comes to the fore [13].

When getting introduced to the course of descriptive geometry, many students face difficulties associated with the representation of simple shapes in space, which further leads to problems understanding the course as a whole. An application developed on the augmented reality platform that displays a three-dimensional model of geometric elements and their projections on the screen of a mobile device in real time can help to improve comprehension and, thereby, make a significant step towards the development of spatial thinking.

With this approach, it is assumed to create a visual educational environment with a group of learning situations in which the development of students’ visual thinking skills is pinpointed. These settings presume the existence of both conventional visual aids as well as specific methods and procedures that enable engaging the activity of vision in order to produce fruitful outcomes [14–16].

As the study showed, the software environments for which the idea of “dynamic geometry”, or interactive geometric systems, had become the leading idea, deserved high praise. “This is an environment that allows you to create dynamic drawings, i.e. computer geometric drawings-models, the source data of which can be varied while preserving the entire construction algorithm, view them and work with them” [17]. The most widespread among them are Chile Cabri 3D (France) and The Geometry’s Sketchpad by Key Curriculum Press (USA).

In the field of mathematics education, proof has been the subject of a great number of studies, all of which have come to the conclusion that it is both important and useful [18]. Despite this consensus, the issue is still considered one that warrants more investigation. Therefore, a widespread agreement has not been established on the coherent theoretical basis for mathematical reasoning that a number of scholars [19] consider essential.

B. Augmented Reality in Education

Augmented reality (AR) is a computer technology that allows the user to see the real world with virtual objects superimposed on it, by creating the effect of their presence in a single space. The terms virtual reality and augmented reality are often erroneously equated, since virtual reality technologies completely immerse the user in an artificial environment, not allowing to perceive the real world around him. There are two main principles of building augmented reality: (a) based on a marker and (b) based on the user’s coordinates [20].

Marker-free technologies are often used in mobile devices using various built-in sensors. A marker is an item that is situated in the surrounding area and is examined by specialized software in order to facilitate the depiction of virtual objects in later stages [21]. The computer may project a virtual item onto the marker, which will imitate the impact of its existence in the surrounding region. This information is gleaned from the location of the marker in space, which is obtained via the use of a camera by the program. Under the condition that high-quality camera models and extra visual filters are used, it is possible to accomplish the effect of making a virtual item seem almost genuine and authentic to the interior space around it. The marker is often used to create a unique picture, which is then pasted onto a piece of paper. Drawing requirements might vary substantially depending on the image recognition method being used since various algorithms demand different forms of drawing. Markers can also be three-dimensional figures and even people’s eyes and faces [22].

Augmented reality technology has a wide range of possibilities of use in education. It is assumed that the use of augmented reality technology in educational institutions in the framework of subjects such as mathematics and geometry will promote the teaching of the subject to another level. One particular advantage of using this technology in education is that it presents excellent visualization capabilities [23].

Based on the main characteristics of AR systems and the principles of augmented reality construction, we have analyzed software systems that could be used in training through the organization of gaming activities. With the help of these systems, you can create interactive 3D objects, discovering the fascinating world of geometry. As a result, the user has a printed set of spatial geometry problems with markers. When launching the application, you need to use a smartphone video camera. The student points the camera at the task card so that a 3D object is displayed on the screen, which is a visualization of the corresponding task from the textbook [24].

The study carries substantial significance for the educational domain. Educators benefit from a robust tool that transforms abstract geometrical concepts into interactive experiences, enriching lesson delivery. This interactive modality enhances students’ comprehension by converting theoretical concepts into tangible, manipulable objects, fostering deeper understanding and engagement. Such dynamic visualization tools elevate learning experiences, making abstract geometric concepts tangible and digestible.

Educational Institutions stand to gain by integrating modern, tech-driven pedagogy, positioning themselves as progressive in teaching methodologies. This not only elevates their academic stature but also attracts tech-savvy enrollees seeking modern learning environments.

EdTech Developers receive invaluable feedback, refining their software to better cater to educational needs, ensuring optimal utility and market relevance.

In essence, this research underscores the pivotal role of dynamic geometry software in revolutionizing plane geometry education. By bridging traditional teaching with technology, it promises enriched learning experiences, potentially reshaping the pedagogical landscape of geometry instruction.

Thus, AR can become a part of education, thanks to technologies that support flexible, affordable, individual learning, and will make education more qualitative, effective and visually compelling.

III. METHODOLOGY

In the course of the research, we developed and implemented an elective course on geometry for 10th grade students "Geometric constructions on a plane and in space", within which all information is presented visually using visualization tools. The purpose of the elective course was to deepen and expand students' knowledge on the topic "Geometric constructions". To achieve this goal, the following tasks were solved in the learning process: development of students' thinking (spatial, practical, logical); enhancement of students' constructive skills; formation of readiness to apply the acquired knowledge in solving practical and research tasks [25]. For the effective use of the developed course with the use of visualization tools, we conducted an experimental work on the basis of the school of Shymkent, Kazakhstan. There are 42 students, who studied in 9th grade, participated in the experiment.

A. Research Design and Approach

This study is anchored in a mixed-methods design, which strategically combines qualitative and quantitative research methodologies. By utilizing both techniques, we ensure a multi-dimensional analysis that offers a richer understanding of the research topic. Through this amalgamation, the research aims to provide a holistic assessment of how dynamic geometry environment software and augmented reality tools impact geometry education. This integrated approach empowers us to capture both statistical trends and nuanced individual experiences, thereby ensuring depth and breadth in our findings.

B. Data Analysis

Quantitative Analysis: Data from questionnaires were statistically analyzed using SPSS software (Version 26). Descriptive statistics, t-tests, and ANOVAs were employed to discern patterns and correlations.

Qualitative Analysis: Transcripts from focus group discussions and interviews underwent thematic analysis, employing NVivo software (Version 12). This enabled the identification of recurrent themes and nuances pertaining to the adoption and efficacy of technology in geometry education.

C. Reliability and Validity

To uphold the reliability of our research instruments, we conducted a pilot test of the questionnaire on a select group, refining questions based on their feedback to remove ambiguity. Further, the validity of the findings was enhanced through the triangulation of qualitative components with the quantitative data. This rigorous methodology ensures that the results are both consistent and accurately representative of the broader context under investigation. Through these measures, the study aims to maintain a high standard of academic rigor and trustworthiness in its conclusions.

D. Utilizing Augmented Reality for Enhanced Geometry Education

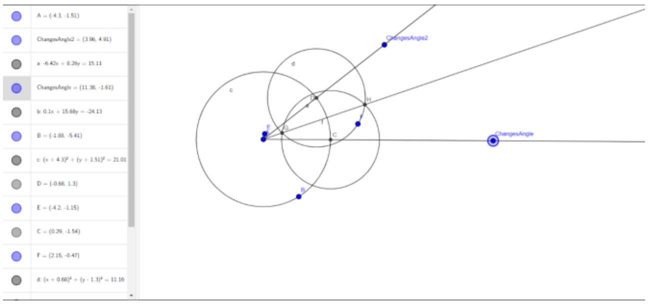
In the subject of planimetry, the visualization and mixing of digital and actual objects provide an opportunity for development of a novel approach to the solution of issues. After analyzing the problem of solving geometry problems in the plane by schoolchildren and the possibilities of using augmented reality for visualization in teaching, it was concluded that the use of this technology in this area has promising prospects. Therefore, a mobile application was developed since mobile devices are now considered as the most widespread and easily accessible kind of device. The following examples of technologies and tools have been used in the development of the proposed application:

- marker technology of augmented reality;
- a mobile device is used as a display device;
- a mobile device is used as an auxiliary tool in solving stereometric problems from the school geometry course.

The developed software application can be employed as an auxiliary tool for solving geometry problems within the framework of a visual approach to teaching mathematics.

The user has a printed set of geometry problems in the plane with markers (augmented reality labels). When launching the application on a mobile device, the capture of the video camera should be enabled. The user points the camera at the task so that the marker is in the field of view of the webcam, a three-dimensional object attached to the label is displayed on the screen, representing a visualization of the task, for example, depicts a section of a pyramid in accordance with the data of the current task.

Fig. 1 demonstrates an example of applying a dynamic geometry environment for the task "Bisect an angle". Fig. 2 illustrates the same example when students change the angles, sizes of circles, and location of points in real time. In own case, it allows to students to consider different cases of one task and understand plane geometry elements deeply and better imagine the given task.



$A = (-4.3, -1.51)$

$ChangesAngle2 = (3.96, 4.91)$

$a: -6.42x + 8.26y = 15.11$

$ChangesAngle = (11.38, -1.61)$

$b: 0.1x + 15.68y = -24.13$

$B = (-1.88, -5.41)$

$c: (x + 4.3)^2 + (y + 1.51)^2 = 21.01$

$D = (-0.68, 1.3)$

$E = (-4.2, -1.15)$

$C = (0.29, -1.54)$

$F = (2.15, -0.47)$

$d: (x + 0.68)^2 + (y - 1.3)^2 = 11.16$

Fig. 1. Bisect an angle task.

As part of the course, multimedia lessons on the topics studied were constructed using visualization tools such as time tape, QR codes, intelligence maps, infographics, three-dimensional graphics, virtual and augmented reality (VR and AR).

The use of active forms of classes, project and research activities with the use of visualization tools, contributed to the successful implementation of educational tasks, the development of cognitive activity, independence. The elective course program is designed for 20 hours and contains material that expands the information of the school geometry course on constructions on the plane and in space. Table I demonstrates the course program that was developed integrating dynamic learning environment.

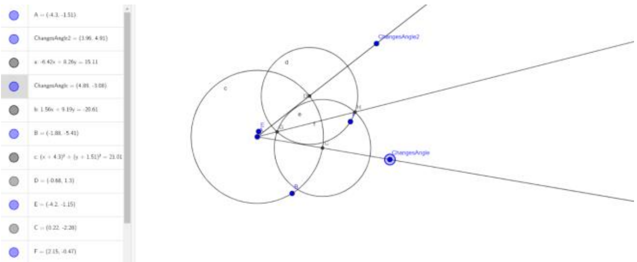


Fig. 2. Bisect an angle in a dynamic geometry environment.

TABLE I: COURSE PROGRAM THAT WAS DEVELOPED INTEGRATING DYNAMIC LEARNING ENVIRONMENT

Topic of classes	Hours	Visualization tools	Student activities
Elementary geometric constructions. Application of elementary constructions in solving problems.	1	QR Code	Input diagnostics, a workshop on problem solving.
Methods for solving problems for building on a plane. 1. The Geometric place of points method 2. The method of geometric transformations (symmetry, rotation, similarity method). 3. The algebraic method 4. Inversion method	8	Intelligence maps, infographics	Research, individual search, heuristic, independent work.
Methods for solving problems on constructions in space. 1. The axiomatic method. 2. Track method. 3. The method of matching points	8	Three-dimensional graphics, virtual and augmented reality.	Solving research tasks, doing homework, preparing for project activities.
Lobachevsky geometry models	1	Infographics, time feed, three-dimensional graphics, virtual and augmented reality.	Model development
Final lesson	2	Infographics, three-dimensional graphics, virtual and augmented reality.	Project protection

E. Formulation of Research Hypotheses

In the evolving landscape of pedagogical research, the role of distinct teaching methodologies and their impacts on student performance remains a focal point of inquiry. Within this context, the present study endeavors to scrutinize the efficacy of two instructional approaches by analyzing student outcomes on a series of construction tasks. Specifically, the research postulates the following hypotheses:

Hypothesis I.

H0: Students from both groups (Experimental group and control group) will obtain equal number of points from Construction Task 1.

H1: Students from both groups (Experimental group and control group) will not obtain equal number of points from Constructions Task 1.

Hypothesis II.

H0: Students from both groups (Experimental group and control group) will obtain equal number of points from Construction Task 2.

H1: Students from both groups (Experimental group and control group) will not obtain equal number of points from Construction Task 2.

Hypothesis III.

H0: Students from both groups (Experimental group and control group) will obtain equal number of points from Construction Task 3.

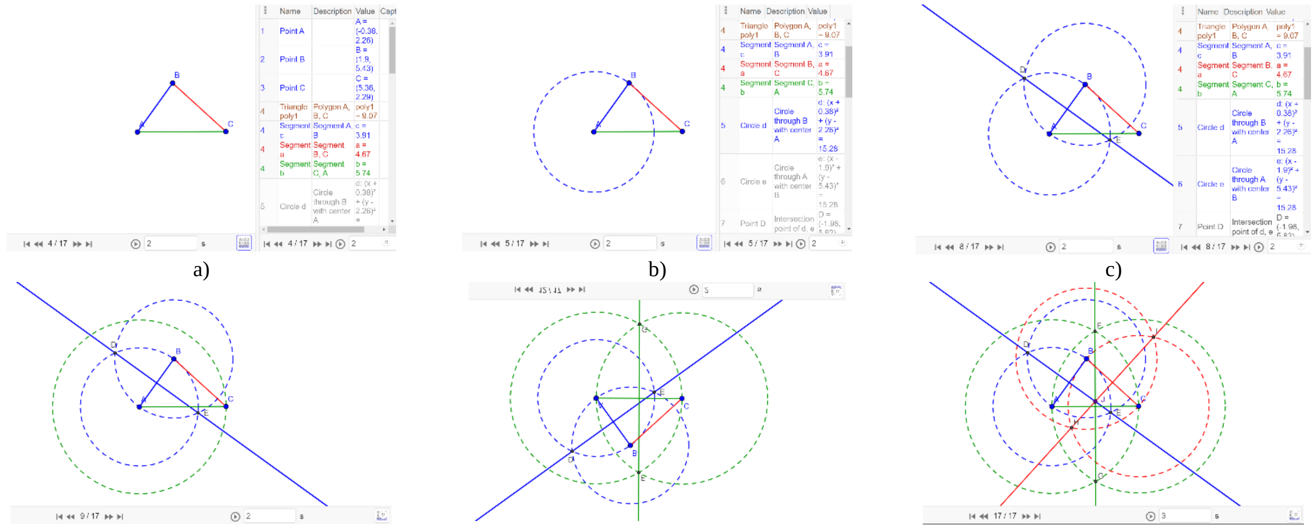
H1: Students from both groups (Experimental group and control group) will not obtain equal number of points from Construction Task 3.

IV. DYNAMIC GEOMETRY SOFTWARE IN USE

The elective course "Geometric constructions on the plane and in space" contained both theoretical and practical parts. When studying the topics of the course, knowledge of construction methods was deepened, the geometric places of points on the plane and in space were scrutinized in detail, the inversion method and the axiomatic method were studied. In the process of practical work concerning solution of geometric problems, students used visualization tools. After each topic, students' achievements were evaluated, diagnostic sections were carried out, and test tasks were offered. At the end of the course, the defense of projects that students performed in microgroups was organized. The students were given a task requiring integrated knowledge on the topics being studied using visualization tools. Fig. 3 shows an example of using dynamic geometry software to study problems of plane geometry.

In the study, the QR code served as a medium to encapsulate details pertinent to the recurrence of basic planar geometric constructions and supplementary data for autonomous tasks. This encoded approach facilitated streamlined access to foundational geometric principles and supplementary resources for student-initiated exploration. Concurrently, comparative infographics were employed as an illustrative tool, offering a visual juxtaposition of the spatial positioning of points in both planar and three-dimensional contexts during the execution of project-based assignments. Through this graphical representation, intricate geometric relationships and variances were rendered more comprehensible and discernible to the learners.

The involvement of intelligence maps in the study of geometric transformations methods made it possible to present information about the particular method in a form that requires minimal time and resources for its perception, analysis and understanding. The time tape was used when considering issues related to Lobachevsky geometry models, as well as in the process of creating a chronology of work on projects [26]. During the formative experiment, students were shown the developed 3D models of Lobachevsky geometry, virtual and augmented reality (VR and AR) technologies were used, which contributed to a better understanding of the three-dimensional shape of surfaces by students.



a) b) c)

d) e) f)

Fig. 3. Explore the construction of the circumcenter and its relationship with the triangle; a) Construction of a triangle by three points; b) Constructing a circle with the center A by the radius AB; c) Constructing a circle with the center B by the radius AB; d) Constructing a circle with the center A by the radius AC; e) Constructing a circle with the center C by the radius AC; f) Constructing circles with the centers B and C by the radius BC.

V. RESULTS

In the experimental class (10 “B”), the elective course was taught using various visualization tools: time tape, QR code, intelligence map, infographics, 3D modeling and Augmented Reality (AR). In contrast, another experimental class (10 “A”) students were presented with material without these funds. As part of the study, the level of development of constructive skills of schoolchildren was revealed. In the end of the course, we gave three tasks for an exam and research hypothesis to explore students’ contribution and results.

At the conclusion of the semester, each student received both a test and a questionnaire that they were required to fill out. The examination consisted of three separate planimetric building assignments, each of which had a varying degree of difficulty. In addition, students were issued an anonymous questionnaire. This ensured that responders would remain anonymous, and since students were aware of the outcomes of the examination, they provided genuine responses to the questions included in the questionnaire.

Table II demonstrates the participants' responses to the anonymous questionnaire. It is noteworthy that for students, traditional lessons are still necessary and essential as they often applied their notes for preparation prior to the final examination. In our pedagogical experiment, all the participated students answered to the questionnaire and near 55% students used dynamic geometry environment. Most of them did not need to search and use alternative materials, only 10.5% of students addressed to external source of materials.

TABLE II: ANSWERS OF PARTICIPANTS FOR THE QUESTIONNAIRE

Question	Yes	No
Was the traditional model useful?	89.47%	10.53%
Was the assembling of models difficult?	54,92%	45,08%
Did the dynamic geometry environment software help you to better understand construction problems in plane geometry?	89,62%	10,38%

Making paper models may seem like a “step back” in the modern world of computer simulations, but our students really benefited from them, too. Control group students found it to be really intriguing if we built these models during the lecture, and the group dynamic was wonderful. However, 55% of students said that they had trouble putting the model together. The majority of students believe that using these models helped them comprehend the construction tasks better. We established test hypotheses that were studied on the data obtained using the Wilcox test as normality of the data was discarded by Tests of Assumptions.

TABLE III: WILCOXON TEST FOR THE CONSTRUCTION TASK 1

Yes	Convergence with continuity correction			Convergence without continuity correction		
	Z-value	Probability Level	Reject the hypothesis H0	Z-value	Probability Level	Reject the hypothesis H0
Median = 0	3.7509	0.000179	1	3.7208	0.000207	1
Median <0	3.7509	0.999793	0	3.7691	0.999897	0
Median >0	3.7509	0.000091	1	3.7213	0.000102	1

Thus, according to Table III, we do not accept the zero H_0 and will instead go with the alternative hypothesis. Experimental group students were better than control group students in Task 1 of the final exam. Their average point achievement increased by 9.5%.

TABLE IV: WILCOXON TEST FOR THE CONSTRUCTION TASK 2

Yes	Convergence with continuity correction			Convergence without continuity correction		
	Z-value	Probability Level	Reject the hypothesis H0	Z-value	Probability Level	Reject the hypothesis H0
Median != 0	3.7512	0.000198	1	3.6911	0.000219	1
Median <0	3.7512	0.999799	0	3.7292	0.999913	0
Median >0	3.7512	0.000112	1	3.6906	0.000109	1

Thus, according to Table IV, we do not accept the H_0 and will instead go with the alternative hypothesis. Experimental group students were better than control group students in Task 2 of the final exam. Their average point achievement increased by 13.7%.

TABLE V: WILCOXON TEST FOR THE CONSTRUCTION TASK 3

Yes	Convergence with continuity correction			Convergence without continuity correction		
	Z-value	Probability Level	Reject the hypothesis H0	Z-value	Probability Level	Reject the hypothesis H0
Median != 0	3.7709	0.000212	1	3.7068	0.000216	1
Median <0	3.7709	0.999879	0	3.7289	0.999911	0
Median >0	3.7709	0.000103	1	3.6917	0.000112	1

Thus, according to Table V, we don’t accept the zero hypothesis and will instead go with the alternative hypothesis. On the final test, students in the experimental group performed significantly better than students in the control group in Task 3. They had a 13.7% improvement in their overall point attainment average.

Table VI demonstrates overall results of the students including their results during the semester and final exam. In the control class, the average and high level of development of constructive skills increased by 10% in comparison with the result obtained at the control stage of the experiment.

TABLE VI: RESULTS OF CONTROL DIAGNOSTICS OF 10TH GRADE STUDENTS

Knowledge Levels	At the beginning of the experiment		At the end of the experiment		Dynamics	
	Control group results	Experimental group results	Control group results	Experimental group results	Control group results	Experimental group results
Low	8	9	6	3	10%	27%
Average	9	8	10	12	5%	18%
High	3	5	4	7	5%	9%

In Table VII, a comprehensive comparison is presented between the results of the current investigation and those obtained from state-of-the-art research. Evidently, there is an escalating interest in the utilization of both virtual and augmented reality as effective mediums in the learning process across various subjects. Such innovative techniques have been observed to be implemented in diverse educational spheres including human anatomy, English language acquisition, and geographical studies. However, a notable limitation in the contemporary studies pertains to their singular reliance on a specific tool, predominantly mobile phones, for delivering virtual and augmented reality content. Contrarily, our study distinguishes itself by endorsing a versatile approach, enabling the use of a myriad of devices such as mobile phones, personal computers, and tablets. This flexibility potentially augments the adaptability and accessibility of our proposed method, enriching the overall learning experience.

TABLE VII: PREVIOUS RESEARCH ABOUT VIRTUAL REALITY

Study	**Subject**	**Learning type**	**Technology**	**Tool**
Proposed study	Geometry	Practice Based Learning	Augmented Reality	Mobile Phone, Tab, Personnel Computer
Taufiqurrohman & Sumbawati, 2020 [27]	Geography	Project Based Learning	Virtual Reality	Mobile Phone
Ahmad *et al*., 2022 [28]	Anatomy	Practice Based Learning	Augmented Reality	Mobile Phone
Saepuloh & Salsabila, 2022 [29]	English vocabulary	Experience Based Learning	Virtual Reality	Personnel Computer
Arsadhana *et al*., 2022 [30]	Early childhood and students with special needs	Experience Based Learning	Virtual Reality	Mobile Phone

Within the experimental cohort, there was a notable enhancement of 27% in the average to high-level development of constructive skills relative to the outcomes at the control juncture. The findings from this empirical investigation underscore the efficacy of incorporating visualization tools during geometry instruction. Such results bolster the proposition that visual aids can significantly optimize learning outcomes in geometry. This elevation in performance accentuates the potential of tailored pedagogical tools in enhancing student competencies.

VI. DISCUSSION

A. Impact of Dynamic Geometry Environments on Technologically Enhanced Geometry Learning

The purpose of this research is to investigate the possibilities of learning geometry in the contexts that have been technologically upgraded, in particular, situations that incorporate dynamic and visual representations of geometry. The findings of this research provide more insight into the influence that visual and sensory-motor skills have on the learning of geometry within school settings. According to the findings of the pilot test, the experimental group performed better overall than the control group did in all of the areas. The fact that the experimental group performed much better than the control group on the "advanced" test provides an answer to the study question by indicating that the dynamic geometry environment technology had a more significant impact on higher levels of geometry learning. These findings provide further evidence that learning plane geometry through the use of online technologies that are displayed on a screen is beneficial. This is the case not only in terms of mental rotation, but also in terms of the relations between the various components of geometric figures and the performance of informal logical reasoning with the properties. Even though three-dimensional ideas are displayed on a two-dimensional computer screen, the students were still successful in achieving the learning outcomes for geometry that were set for them by using an inquiry-based form of learning that was supplemented with the manipulability of a dynamic geometry environment [31].

B. Embodied Cognition in 3D Geometry Learning

According to the findings of the current research, both digital and hands-on learning settings boost knowledge acquisition and permit proper retention of the material when it comes to learning about 3D geometry. These findings provide credence to the embodied cognition viewpoint as well as a strategy to learning mathematics that emphasizes hands-on experience. The embodied cognition hypothesis asserts that the location of cognition is in the somatic and tactile interactions that we have with the physical environment.

Despite the fact that some problems, such as the degree of students' ability for mathematical proof and reasoning, haven't been solved yet [32, 33], the international recognition of the significance of proof in the process of learning mathematics has generated a substantial amount of study. It has been noticed that students do not sense the need for evidence and are able to separate confirmation from explanation or proof [34]. This makes the challenges students face in higher education, which places a strong focus on formal criteria such as rigor and logic, far more challenging [35].

Students in elementary and secondary education should focus most of their study on evidence and reasoning in the field of geometry [36]. To be more explicit, it has been shown that geometric modifications play an essential role in the identification of similar figures [37]. Students gain the ability to defend and verify their ideas as well as develop relationships with algebra when they investigate the attributes of forms and the pictures of those shapes via the use of geometric transformations [38].

C. Dynamic Geometry Environments and Deductive Reasoning

Dynamic geometry environments, on the other hand, have been shown in a number of empirical investigations to be more beneficial to the development of deductive reasoning. In spite of the fact that proof assistant software has been widely used in recent years and there is ample evidence of learning how to solve proof problems, the value of this software for classroom student learning has not been conclusively proved [39–41]. In the context of geometric transformations and the utilization of various types of illustration, Gamification needs to accommodate structures that can be interactively modified by having to drag completely separate objects. Thus, the program is granted a huge potential for proof and visualization in the context of geometric problems. When working with dynamic geometry software to solve proof problems, drag functions are ideal for doing the different construction tasks in plane and spatial geometry. Students who are provided with education on the concepts and congruency qualities of triangles may find it simpler to learn deductive procedures when it comes to proof challenges. The dragging feature that is incorporated into dynamic geometry software may be used to do an analysis of the qualities that are preserved on the grounds of resemblance. Moving the figures about and dragging one of the vertices to make a full family offers a huge amount of potential, but it is unclear whether or not the students understand the notion of invariability or the various benefits that are involved [42]. One of the possible benefits is the establishment of a boundary case between condition-compliant instances and examples that do not comply with the condition, which is essential for generalizing conjectures [43]. Students could be encouraged to approach issues in a more logical manner [44] if they are aware that not all situations can be validated, even using dynamic geometry software. The development of more comprehensive proofs may be helped by the possibility of using visual representations simultaneously with algebraic formulas [45].

VII. CONCLUSION

The provision of high-quality resources for independent study is an essential component of each and every lesson, as well as the effective use of the dynamic learning environments. The process of independent study is given a fresh boost by the open source learning management systems' support for the dynamic geometry learning environment. The issue that has to be answered is how the dynamic learning environments can be useful in plain geometry construction tasks. Here, we presented an introduction to the idea of materials for the dynamic learning instruments, focusing in particular on the materials used for paper models and anaglyph models of the same jobs. These resources are received quite well by students, and the majority of them understand that they are meant to serve as a refreshment for the lecture. Participants in the survey were students enrolled in a Constructional Geometry class. These students have a high level of intrinsic drive, and this course is seen as being highly helpful and essential for the job that they will be doing in the future.

Following the completion of the pilot testing, we discovered that the average and high level of development of constructive skills increased by 10% in comparison with the result obtained at the control stage of the experiment. Moreover, in the experimental class, the average and high level of development of constructive skills increased by 27% in comparison with the result obtained at the control stage of the experiment. These findings serve as inspiration for us as we go on with our work since we would want to test the largest group possible and increase the scope of our materials.

The results of the study, while promising, have certain limitations. Firstly, the applicability of the dynamic geometry software may be constrained by the technological infrastructure available in various educational settings, potentially leading to disparities in access and outcomes. Moreover, the study might not have covered the entire spectrum of learners, particularly those with specific learning disabilities or those who might find visual learning counterintuitive. It's also pertinent to note that while software provides dynamic visualization, the effectiveness of learning largely depends on the pedagogical strategies employed by educators.

For future research, it would be beneficial to (a) investigate the adaptability of the software in diverse educational settings, from urban to rural, and for varied student demographics; (b) examine the long-term retention and application of learned concepts through dynamic geometry tools compared to traditional methods; (c) explore potential modifications to the software to cater to students with specific learning needs; (d) study the training requirements and adaptability of educators in effectively integrating such software into their teaching regimen.

In summary, while the current research paves the way for innovative teaching methods in plane geometry, future inquiries should delve deeper into its broad applicability, inclusivity, and long-term impact.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

ET contributed to all parts of the research paper. NM has been involved in pedagogic experiments, data gathering, and analysis. Both authors contributed to the article and approved the submitted version.

REFERENCES

[1] E. Campos-Pajuelo, L. Vargas-Hernandez, F. Sierra-Liñan, J. Zapata-Paulini, and M. Cabanillas-Carbonell, "Learning the chemical elements through an augmented reality application for elementary school children," *Advances in Mobile Learning Educational Research*, vol. 2, no. 2, pp. 493–501, 2022. https://doi.org/10.25082/AMLER.2022.02.018

[2] K. Lavidas, K., Z. Apostolou, Z., and S. Papadakis, "Challenges and opportunities of mathematics in digital times: Preschool teachers' views," *Education Sciences*, vol. 12, no. 7, pp. 1–12, 2022. https://doi.org/10.3390/educsci12070459

[3] M. Turgut, "Reinventing geometric linear transformations in a dynamic geometry environment: Multimodal analysis of student reasoning," *International Journal of Science and Mathematics Education*, vol. 20, no. 6, pp. 1203–1223, 2022. https://doi.org/10.1007/s10763-021-10185-y

[4] M. Turgut, J. Smith, and C. Andrews-Larson, "Symbolizing lines and planes as linear combinations in a dynamic geometry environment," *The Journal of Mathematical Behavior*, vol. 66, 100948, 2022. https://doi.org/10.1016/j.jmathb.2022.100948

[5] S. Gökçe and P. Güner, "Dynamics of GeoGebra ecosystem in mathematics education," *Education and Information Technologies*, vol. 27, no. 4, pp. 5301–5323, 2022. https://doi.org/10.1007/s10639-021-10836-1

[6] N. Dahal, N. Manandhar, L. Luitel, B. Luitel, B. Pant, and I. Shrestha, "ICT tools for remote teaching and learning mathematics: A proposal for autonomy and engagements," *Advances in Mobile Learning Educational Research*, vol. 2, no. 1, pp. 289–296, 2022. https://doi.org/10.25082/AMLER.2022.01.013

[7] K. Li, Y. Zhang, K. Li, Y. Li, and Y. Fu, "Image-text embedding learning via visual and textual semantic reasoning," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 45, no. 1, pp. 641–656, 2022. 10.1109/TPAMI.2022.3148470

[8] N. Emul, H. Gulkilik, and H. Kaplan, "Pre-service mathematics teachers' experience with a dynamic geometry environment whilst reasoning in relation to locus problems: A detailed look at strategies," *Computers in the Schools*, vol. 39, no. 4, pp. 297–322, 2022. https://doi.org/10.1080/07380569.2022.2075165.

[9] S. Çildir and A. Şen, "Investigation of the GeoGebra-supported teaching material development process of pre-service physics teachers," *Balıkesir Üniversitesi Fen Bilimleri Enstitüsü Dergisi*, vol. 25, no. 1, pp. 90–106, 2023. https://doi.org/10.25092/baunfbed.1126834

[10] Y. Liu, Y. Wei, H. Yan, G., Li, and L. Lin, L, "Causal reasoning meets visual representation learning: A prospective study," *Machine Intelligence Research*, vol. 19, no. 6, pp. 485–511, 2022. https://doi.org/10.1007/s11633-022-1362-z

[11] T. Dana-Picard and S. Hershkovitz, "Exploration of polygons in a STEAM framework: technology and cultural background," *International Journal of Mathematical Education in Science and Technology*, vol. 54, no. 6, pp. 1112–1131, 2022. https://doi.org/10.1080/0020739X.2022.2073280

[12] L. Jiao, J. Chen, F. Liu, S. Yang, C. You, X. Liu, and B. Hou, "Graph representation learning meets computer vision: A survey," *IEEE Transactions on Artificial Intelligence*, vol. 4, no. 1, pp. 2–22, 2022. 10.1109/TAI.2022.3194869

[13] O. Birgin and H. Acar, "The effect of computer-supported collaborative learning using GeoGebra software on 11th grade students' mathematics achievement in exponential and logarithmic functions," *International Journal of Mathematical Education in Science and Technology*, vol. 53, no. 4, pp. 872–889, 2020. https://doi.org/10.1080/0020739X.2020.1788186.

[14] C. Markopoulos, P. Bruck, and K. Petta, "Exploring Students' Geometrical Thinking Through Dynamic Transformations Using 3D Computer-Based Representations," *Journal of mathematics education at Teachers College*, vol. 13, no. 2, pp. 1–7, 2022. https://doi.org/10.52214/jmetc.v13i2.9009.

[15] S. Beltozar-Clemente, F. Sierra-Liñan, J. Zapata-Paulini, and M. Cabanillas-Carbonell, "Augmented reality mobile application to improve the astronomy teaching-learning process," *Advances in Mobile Learning Educational Research*, vol. 2, no. 2, pp. 464–474, 2022. https://doi.org/10.25082/AMLER.2022.02.015

[16] E. Gelova and M. Vitanova, "Solving tasks from the topic plane equation using GeoGebra," *Balkan Journal of Applied Mathematics and Informatics*, vol. 5, no. 2, pp. 17–25, 2022. https://js.ugd.edu.mk/index.php/bjami/article/view/5189

[17] T. Karakose, H. Polat, R. Yirci, T. Tülübaş, S. Papadakis, T. Ozdemir, and M. Demirkol, "Assessment of the relationships between prospective mathematics teachers' classroom management anxiety, academic self-efficacy beliefs, academic amotivation and attitudes toward the teaching profession using structural equation modelling," *Mathematics*, vol. 11, no. 2, p. 449, 2023. https://doi.org/10.3390/math11020449

[18] T. Iwano, D. Pereira, and Scherer, "Teaching Spatial Geometry Through 3D Visualization and Augmented Reality," *In Methodologies and Use Cases on Extended Reality for Training and Education*, pp. 185–214, IGI Global, 2022. 10.4018/978-1-6684-3398-0.ch008

[19] J. Devagiri, S. Paheding, Q. Niyaz, X. Yang, and S. Smith, "Augmented Reality and Artificial Intelligence in industry: Trends, tools, and future challenges," *Expert Systems with Applications*, vol. 207, 118002, 2022, https://doi.org/10.1016/j.eswa.2022.118002.

[20] R. Sousa and F. Alves, "Didactic engineering and learning objects: A proposal for teaching parabolas in analytical geometry," *Indonesian*

Journal of Science and Mathematics Education, vol. 5, no. 1, pp. 1–16, 2022. http://dx.doi.org/10.24042/ijsme.v5i1.11108

[21] C. Cumino, M. Pavignano, M. Spreafico, and U. Zich, "Geometry to build models, models to visualize geometry," *Digital Experiences in Mathematics Education*, vol. 7, no. 1, pp. 149–166, 2021. http://dx.doi.org/10.24042/ijsme.v5i1.11108

[22] Z. Kovács, T. Recio, P. Richard, S. Van Vaerenbergh, and M. Vélez, "Towards an ecosystem for computer-supported geometric reasoning," *International Journal of Mathematical Education in Science and Technology*, vol. 53, no. 7, pp. 1701–1710, 2020. https://doi.org/10.1080/0020739X.2020.1837400

[23] O. Ng and H. Ye, "Mathematics learning as embodied making: primary students' investigation of 3D geometry with handheld 3D printing technology," *Asia Pacific Education Review*, vol. 23, no. 2, pp. 311–323, 2022. https://doi.org/10.1007/s12564-022-09755-8

[24] S. Avcu, and B. Çetinkaya, "An instructional unit for prospective teachers' conceptualization of geometric transformations as functions," *International Journal of Mathematical Education in Science and Technology*, vol. 52, no. 5, pp. 669–698, 2018. https://doi.org/10.1080/0020739X.2019.1699966

[25] M. Futai, T. Bittencourt, H. Carvalho, and D. Ribeiro, "Challenges in the application of digital transformation to inspection and maintenance of bridges," *Structure and Infrastructure Engineering*, vol. 18, no. 10–11, pp. 1581–1600, 2022. https://doi.org/10.1080/15732479.2022.2063908

[26] O. Semenikhina, V. Proshkin, and O. Naboka, "Application of Computer Mathematical Tools in University Training of Computer Science and Mathematics Pre-service Teachers," *International Journal of Research in E-learning*, vol. 6, no. 2, pp. 1–23, 2020. https://doi.org/10.31261/IJREL.2020.6.2.06

[27] T. Taufiqurrohman, and M. Sumbawati, "Penerapan Media Virtual Tour Dengan Google Expedition Dalam Pembelajaran Project Based Learning di SMK Negeri 10 Surabaya," *IT-Edu : Jurnal Information Technology and Education*, vol. 5, no. 1, pp. 247–253, 2020. https://ejournal.unesa.ac.id/index.php/it-edu/article/view/37483

[28] I. Ahmad, S. Samsugi, and Y. Irawan, "Penerapan Augmented Reality Pada Anatomi Tubuh Manusia Untuk Mendukung Pembelajaran Titik Titik Bekam Pengobatan Alternatif," *Jurnal Teknoinfo*, vol. 16, no. 1, 2022. https://doi.org/10.33365/jti.v16i1.1521

[29] A. Saepuloh and V. Salsabila, "Virtual Reality Dalam Pengajaran Kosakata Bahasa Inggris Materi Things Around," *JE (Journal of Empowerment)*, vol. 3, no. 1, pp. 2–22, 2022. https://doi.org/10.35194/je.v3i1.2387

[30] U. Usanto, N. Sucahyo, W. Warta, S. Khie, and I. Fitriyani, "Transformasi Kepemimpinan Yang Bersifat Profetik Dan Pemberdayaan Masyarakat Di Era Society 5.0 Yang Berkelanjutan," *Community Development Journal: Jurnal Pengabdian Masyarakat*, vol. 4, no. 2, pp. 5287–5301, 2023. https://doi.org/10.31004/cdj.v4i2.16607

[31] D. Sultan, B. Omarov, Z. Kozhamkulova, G. Kazbekova, L. Alimzhanova, A. Dautbayeva, and R. Abdrakhmanov, "A–n," *Computers, Materials & Continua*, vol. 74, no. 3, pp. 5625–5640. https://doi.org/10.32604/cmc.2023.033682

[32] N. Ruiz-López, "The instrumental genesis process in future primary teachers using Dynamic Geometry Software," *International Journal of Mathematical Education in Science and Technology*, vol. 49, no. 4, pp. 481–500, 2018. https://doi.org/10.1080/0020739X.2017.1377302

[33] M. Vita, L. Verschaffel, and J. Elen, "Towards a better understanding of the potential of interactive whiteboards in stimulating mathematics learning," *Learning Environments Research*, vol. 21, pp. 81–107, 2018. https://doi.org/10.1007/s10984-017-9241-1

[34] M. Dockendorff and H. Solar, "ICT integration in mathematics initial teacher training and its impact on visualization: the case of GeoGebra," *International Journal of Mathematical Education in Science and Technology*, vol. 49, no. 1, pp. 66–84, 2018. https://doi.org/10.1080/0020739X.2017.1341060

[35] S. Narynov, D. Mukhtarkhanuly and B. Omarov, "Dataset of depressive posts in Russian language collected from social media," *Data in Brief*, vol. 29, 105195, 2020. https://doi.org/10.1016/j.dib.2020.105195

[36] V. Zambak and A. Tyminski, "Examining mathematical technological knowledge of pre-service middle grades teachers with Geometer's Sketchpad in a geometry course," *International Journal of Mathematical Education in Science and Technology*, vol. 51, no. 2, pp. 183–207, 2020. https://doi.org/10.1080/0020739X.2019.1650302

[37] R. Chacón, and M. Estela, "STEAM activities for civil engineering curricula. From Calculus to Digital Twins," *In 2021 IEEE Frontiers in Education Conference (FIE)*, pp. 1–8, 2021. 10.1109/FIE49875.2021.9637163

[38] D. Machado, N. Bastos, A. Hall, and S. Pais, "Volume of geometric solids on the Desmos platform–A didactic experience in Cape Verde," *European Journal of Science and Mathematics Education*, vol. 11, no. 3, pp. 376–391, 2023. https://doi.org/10.30935/scimath/12788

[39] T. B. Bedada and F. Machaba, "The effect of GeoGebra on STEM students learning trigonometric functions," *Cogent Education*, vol. 9, no. 1, 2034240, 2022. https://doi.org/10.1080/2331186X.2022.2034240

[40] G. Murugesan, T. Ahmed, M. Shabaz, J. Bhola, B. Omarov, B., R. Swaminathan, and S. Sumi, "Assessment of mental workload by visual motor activity among control group and patient suffering from depressive disorder," *Computational Intelligence and Neuroscience*, 2022. https://doi.org/10.1155/2022/8555489

[41] T. Dana-Picard and Z. Kovács, "Automated exploration of envelopes and offsets with networking of technologies," *Mathematics in Computer Science*, vol. 17, no. 1, p. 3, 2023. https://doi.org/10.1007/s11786-022-00555-2

[42] A. Puig, I. Rodríguez, J. Baldeón, and S. Múria, "Children building and having fun while they learn geometry," *Computer Applications in Engineering Education*, vol. 30, no. 3, pp. 741–758, 2021. https://doi.org/10.1002/cae.22484

[43] A. Samura, "Improving mathematics critical thinking skills of junior high school students using Blended Learning Model (BLM) in GeoGebra assisted mathematics learning," *International Journal of Interactive Mobile Technologies*, vol. 17, no. 2, pp. 1–23, 2023. https://doi.org/10.1007/s10639-023-12200-x

[44] R. Núñez, C. Galán, and A. Suarez, "Van Hiele and GeoGebra model. An Analysis from Variational Thinking in Basic Education Students," *Journal of Positive Psychology and Wellbeing*, pp. 114–125, 10.1109/FIE49875.2021.9637163.

[45] D. Machado, N. Bastos, A. Hall, and S. Pais, "Volume of geometric solids on the Desmos platform–A didactic experience in Cape Verde," *European Journal of Science and Mathematics Education*, vol. 11, no. 3, pp. 376–391, 2023. https://doi.org/10.30935/scimath/12788