The Effect of Flipped Classroom Learning Model on Students' Understanding of Mathematical Concepts and Higher-Order Thinking Skills

Diana*, Herman Dwi Surjono, and Ali Mahmudi

***Abstract*—Students are required to understand mathematical concepts and higher-order thinking skills as essential mathematic skills in the 21st century. However, empirical evidence found that these two skills were unsatisfactory and still need to be improved. This study develops a flipped classroom learning model by integrating the process of mathematical modeling as a learning stage to support junior high school student's understanding of mathematical concepts and higher-order thinking skills. Besides being designed for online mathematics learning, this model also support face-to-face classroom learning by solving real-life problems using mathematical modeling activities. The research design used a quasi-experimental non-equivalent control group design. The research was conducted in the second semester of the 2021–2022 academic year involving 128 eighth-grade (junior high school) students in Yogyakarta, Indonesia. Students' understanding of mathematical concept and higher-order thinking skills were tested through pre-test and post-test. The results showed that the student's ability to understand mathematical concepts and higher-order thinking skills in the experimental group differed significantly from that of the control group. Thus, the flipped classroom learning model could more effectively increase junior high school students' understanding of mathematical concepts and higher-order thinking skills compared to conventional learning. Other findings state that the flipped classroom learning model could support students' problem-solving abilities and increase student collaboration in learning mathematics.**

***Index Terms*—Flipped classroom, mathematical modeling, understanding of mathematical concepts, higher-order thinking skills**

I. Introduction

The incorporation of technology in school mathematics has posed significant challenges and provided for the teaching and learning, especially during the Covid-19 pandemic. This condition certainly has an impacts on teachers and students readiness for online and offline learning to ensure smooth learning process. Teachers play an essential role in carrying out their duties as creators of the pedagogical environment [1]. Teachers must innovate in developing learning strategies, so students feel interested, motivated, and involved in mathematics.

In the face of the 21st-century, students are required to acquire essential mathematical skills [2]. The Partnership for 21st-century Skills (Partnership 21) is an influential initiative to promote 21st-century skills [3]. These skills include critical thinking, problem-solving, creativity, communication, collaboration, and literacy. This is in line with the formulation of the 21st-century learning paradigm in the 2013 Curriculum in Indonesia, which emphasizes students' ability to find out from various sources, formulate problems, think analytically, cooperate, and collaborate in solving problems [4].

The main goal of learning mathematics is to understand mathematics, including factual, conceptual, and procedural understanding [5]. Understanding mathematics according to Davis [6], includes procedural and conceptual understanding. Procedural understanding is the ability to apply mathematical procedures holistically and precisely to obtain the correct solution. Meanwhile, conceptual understanding is the ability to detect errors, provide logical arguments, and recognize new types of mathematical problems when applying procedures. The study [7] explains that understanding mathematics is related to recognizing relationships between concepts and within concepts, representing concepts in various ways, recognizing relationships between representations, and changing concepts into different representations.

Conceptual understanding can build procedural fluency in teaching mathematics [8]. Understanding concepts is essential to student proficiency [9]. Students with a good understanding of concepts can solve new problems that will be studied in the future and will have a strong desire to be involved in learning mathematics. The importance of understanding mathematics is to help students recognize the importance of their thinking processes and learn from their mistakes. In addition, learning mathematics with understanding and actively constructing new knowledge from experience and previous knowledge will help students to solve new types of problems that will be studied in the future.

Higher-order thinking skills are needed to solve math problems and everyday problems, in addition to understanding concepts. Therefore, higher-order thinking skills are an essential aspect of education in the 21st-century. Higher-Order Thinking Skills (HOTS) in learning mathematics include communication, creativity, problem-solving, and mathematical reasoning [10]. Higher-order thinking skills require a more complex thought process in dealing with or solving a problem [11]. Higher-order thinking skills can prepare students to face challenges in academics, work, and adult responsibilities in

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everyday life [12].

Empirical evidence states that the ability to understand mathematical concepts and students' thinking skills still needs to improve. The ability to absorb national exam questions on geometry and measurement material reaches 58.10% [13]. Students' main obstacles in the exam were the difficulty in understanding the concept of triangular comparisons, the concept of curved length, the concept of internal tangents, prism characteristics, determining the area of a plane, understanding the scope of a solid surface, and understanding the concept of meaning [14]. Furthermore, the ability to understand low mathematical concepts is also shown by the research results [15–18].

Several previous researchers have reviewed and analyzed students' thinking skills [19–21]. However, aforementioned research results have provided information that students' abilities are still at low-level thinking skills. This is reinforced by the results of the 2018 PISA and TIMSS 2015, showing that the math skills of Indonesian students are still low. For example, students have difficulty solving PISA questions because they have difficulty understanding the questions, changing real problems into mathematical form, solving mathematical problems, and interpreting solutions into real results [22, 23]. This is due to the low level of students' mathematical literacy and understanding of the test material [24].

The method of teaching mathematics is the most influential factor that leads to students' difficulties in learning mathematics [25]. Teaching practices that still place mathematics as an abstract subject has required students to require reasoning and creativity and make them accustomed to solving problems [26]. Unfortunately, active learning models are rarely used in learning practice, and mathematics textbooks do not currently promote HOTS for students in Indonesia [27].

Current teaching practices require active and innovative learning models that can change mathematics learning. Learning mathematics should involve the students in order to support students' ability to understand concepts and higher-order thinking skills. In their research, Lee and Lai [28] concluded that the flipped classroom could improve students' higher-order thinking skills. The flipped classroom is one of the newest types of blended learning from recent literature and has been used effectively to support traditional forms of teaching [29, 30]. The flipped classroom is a relevant learning model for facilitating learning during the Covid-19 pandemic. Bergmann and Sams [31] stated that the flipped classroom model is a learning model that aims to increase student activity, cooperation, and support with better time allocation during the learning process.

As a learning model, the flipped classroom has become increasingly popular recently. Baker [32] explained that the flipped classroom includes online components such as learning outside the classroom and active learning in the classroom. The flipped classroom is a new pedagogical method, which uses asynchronous lecturing video where students are given exercises as homework, and active group-based problem-solving activities in class [33]. This model can increase students' enthusiasm for learning material before coming to class [34], and students are highly motivated to achieve learning goals [35]. The learning process in the flipped classroom can improve student self-regulation [36]. Students can watch videos repeatedly until they understand the material and learn at their own pace [37–39].

The design and implementation of flipped classrooms in secondary schools by Lo and Hew [39] have recommended the flipped classroom model for lesson planning, learning outside the classroom, and learning in the classroom. Furthermore, Apriska and Sugiman [40] suggested that applying the flipped classroom model in learning mathematics not only focuses on learning outside the classroom, but also on classroom activities, such as problem-solving activities.

Active learning in the classroom has allowed students to have more time to concentrate for better understanding and application of concepts, such as critical and creative thinking [32]. The flipped classroom model can enhance teamwork in the classroom [41]. Teachers can develop meaningful learning activities to stimulate students to engage in higher-order thinking [42] and higher-order thinking skills are complex thinking, such as evaluating and creating, which allow students to retain information and apply problem-solving solutions for real-world problems in classroom learning [43].

Several researchers have developed and implemented the flipped classroom model in learning mathematics. However, integrating the mathematical modeling process as syntax or steps in the flipped classroom learning model has never been done before. In addition to preparing students for independent learning outside the classroom, this model also focuses on face-to-face learning in class (collaborative class) through real problem-solving activities using mathematical modeling process. The flipped classroom learning model refers to the cycle or process of mathematical modeling by Blum and Leiß [44], namely (1) understanding, (2) simplifying & structuring, (3) mathematizing, (4) working mathematically, (5) interpreting, and (6) validating.

Mathematical modeling can be interpreted as building models from real situations to mathematical models, applying problem-solving processes, or any way that connects the real world with mathematics [45]. Mathematical modeling is more successful in achieving mathematics and problem-solving skills and can improve student learning achievement [46]. Mathematical modeling helps students understand and apply mathematical concepts [47]. Integrating mathematical modeling effectively increases problem-solving performance and reduces students' mathematics anxiety levels [48].

The modeling process starts with a real problem or reality [49]. The types of real problems presented are open-ended and require creativity and persistence. Riyanto and Putri [50] stated that modeling is closely related to higher-order thinking skills (HOTS). Mathematical modeling is a HOTS task, so a mathematical modeling approach must present HOTS-oriented questions. This illustrates the need to develop a flipped classroom learning model by presenting real-world problems in solving problems through a mathematical modeling process to support students' understanding of mathematical concepts and higher-order thinking skills in geometry materials.

II. Method

A. Conceptual Model

The conceptual model framework is designed by integrating mathematical modeling as the syntax of the flipped classroom learning model. The flipped classroom and mathematical modeling concepts reinforce the flipped classroom learning model theory from several experts [32–33, 39, 44]. The designed model consists of three learning stages, namely the planning, implementation, and evaluation stages. The syntax of the flipped classroom learning model has six steps, namely understanding, simplifying/structuring, mathematizing, mathematically working, interpreting, and validating [44]. The conceptual model for developing the flipped classroom learning model is illustrated in Fig. 1.

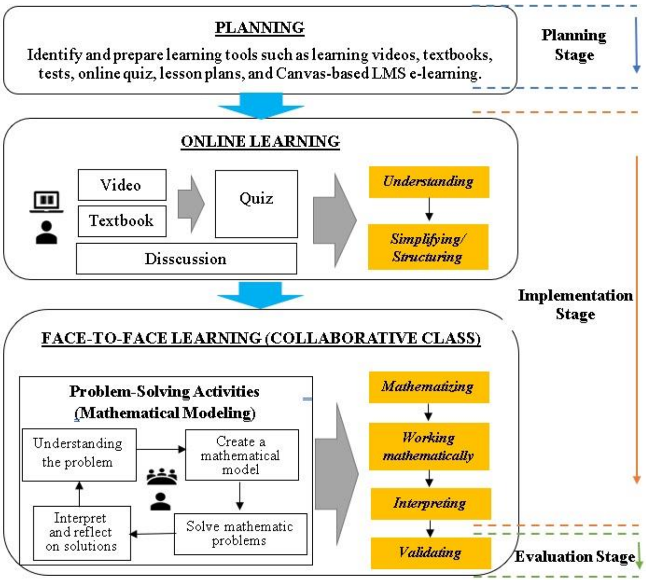

Fig. 1. Conceptual model.

First, the planning stage is to identify and prepare all the tools for learning. Second, the implementation phase includes online and face-to-face learning. Online learning consists of the following steps: (1) understanding, in which students watch videos then note important and difficult points. Students can study textbooks to deepen their understanding. Then, students discuss their understanding with teachers and friends and receive direct feedback through online discussions. (2) simplifying & structuring, in which students complete online quizzes so that teachers can assess and measure students' understanding before entering face-to-face learning in class. Face-to-face learning (collaborative class) consists of the following step: (3) mathematizing, in which the teacher explains the main concepts presented in the video and directs students to form groups. The next step is (4) working mathematically, in which students solve real problems using a mathematical modeling process by discussing and working together in groups, followed by (5) interpreting, where students present the results of group discussion assignments, ask questions, and provide input, and the teacher responds to the results of student discussions. Third, the evaluation stage consists of step (6) validating, in which the teacher reflects on the learning process, concludes the learning material with students, and ends up with an evaluation of learning outcomes.

B. Research Design

The research design used a quasi-experimental non-equivalent control group design, a design that compares the result before and after treatment without assigning participants to the experimental and control groups [51]. In this study, the data were collected using questionnaires, observations, surveys, interviews, rating scales, and tests. The research instruments in questionnaires, observation sheets, rating scale instruments, and tests were validated by experts, then revised to produce valid and reliable research instruments.

This study used a cluster random sampling technique. First, the population was randomly selected based on the district, namely the junior high school in the Pakem sub-district. Afterwards, the samples were randomly selected to determine two experimental and two control classes before coming up with the selected 128 students from two state schools, each of which consisting of two classes, namely the experimental and the control groups. A total of 64 students in the experimental group used the flipped classroom learning model, and 64 students in the control group used a conventional learning approach. The research design is described as follows Table I.

TABLE I: RESEARCH DESIGN [51]

Groups	Pre-test	Treatment	Post-test
Experimental	O_1	X_1	O_2
Control	O_1	X_2	O_2

O_1: Pre-test of Understanding Mathematical Concepts and student HOTS
O_2: Post-test of Understanding Mathematical Concepts and student HOTS
X_1: Flipped Classroom Learning Model
X_2: Conventional Learning

The test on students' understanding of mathematical concept measures 8 grade students or junior high school students' ability to understand mathematical concepts. The team of experts in mathematics education validated this test prior to test implementation. Several indicators of students' understanding of mathematics in detail have been explained by Hoosain [6], among others were recognizing the relationship between concepts and within a concept, representing concepts in different ways, linking between representations of concepts, and turning them into different representations, and followed by the application of mathematical concepts in real life and various situations. The following is a test instrument for mathematical concept understanding presented in Table II.

TABLE II: TEST INSTRUMENT FOR MATHEMATICAL CONCEPT UNDERSTANDING

Indicators	Sub-Indicators	Items
Determine the elements of a concept	Show Explain	1
Recognizing and associating relationships between concepts and within a concept	Solve Attributing Determine	2, 3
Apply mathematic concepts in real life and various situations	Check Criticize	4
	Solve Determine	5

The team of experts in the field of mathematics education conducted test validation based on (1) the suitability of the items and the indicators, (2) the level of difficulty of the items, (3) the use of language according to the correct spelling and (4) the correctness of the concepts used. The mathematical concept understanding test consists of five essay questions. This test was tested on 30 students of the four Pakem public junior high schools. Table III shows that the mathematical concept understanding test is valid where the r_{xy} value is greater than r_{table} 0.361 with 28 degrees of freedom at an alpha level of 0.05. While Cronbach's alpha value is 0.941.

TABLE III: ITEM VALIDITY OF MATHEMATICAL CONCEPT UNDERSTANDING TEST

Items	r_{xy}	r_{table}	Criteria
Item1	0.853	0.361	Valid
Item2	0.919	0.361	Valid
Item3	0.814	0.361	Valid
Item4	0.735	0.361	Valid
Item5	0.908	0.361	Valid

Researchers developed the higher-order thinking skills test to measure students' higher-order thinking skills. The revised Bloom's Taxonomy has six categories from the lowest to the highest: remembering, understanding, applying, analyzing, evaluating, and creating [52]. According to Anderson and Krathwohl [52], HOTS is the ability to think at the highest three levels: analyzing, evaluating, and creating. The test instrument for higher-order thinking skills can be seen in Table IV.

TABLE IV: TEST INSTRUMENT FOR HIGHER-ORDER THINKING SKILL

HOTS Level	Indicators	Items
Analyze	Solve Determine	1
Evaluate	Check Criticize	2, 3
Create	Formulate Designing Produce	4, 5

The expert validated the higher-order thinking skills test and revised it under his guidance and suggestions. The test was tested on 30 students of the one Pakem public junior high school. Table V show that all five items are declared valid because all the r_{xy} item values are greater than r_{table} 0.361. The HOTS test was declared reliable with a Cronbach's alpha value of 0.954 at an alpha significance level of 0.05.

TABLE V: ITEM VALIDITY OF HOTS TEST

Items	r_{xy}	r_{table}	Criteria
Item1	0.894	0.361	Valid
Item2	0.888	0.361	Valid
Item3	0.908	0.361	Valid
Item4	0.782	0.361	Valid
Item5	0.904	0.361	Valid

III. RESULT AND DISCUSSION

Post-tests for the ability to understand mathematical concepts and higher-order thinking skills were given to groups of experimental and control students after learning was completed. Furthermore, the pre-test and post-test data for the experimental and control classes were analyzed using the average, standard deviation, and MANOVA. The pre-test and post-test data have fulfilled normality and homogeneity.

Descriptive statistics were used to compare the average pre-test and post-test scores using the flipped classroom learning model on understanding mathematical concepts and higher-order thinking skills, as presented in Table VI.

Table VI shows that the post-test average score for understanding mathematical concepts in the experimental group (M = 77.03) is higher than the average pre-test score (M = 47.90). Likewise, in the control group, the post-test average score for understanding mathematical concepts (M = 56.17) is higher than the average pre-test score (M = 45.17). The experimental group has an average post-test score of higher-order thinking skills (M = 76.81) higher than the pre-test (46.86). Likewise, the control group has an average post-test score (M = 53.83) higher than the pre-test (M = 44.41). Thus, there is a difference in the average test scores between the understanding of mathematical concepts and students' high-order thinking skills in the experimental and control groups.

TABLE VI: DESCRIPTIVE ANALYSIS OF MATHEMATICAL CONCEPT UNDERSTANDING AND HOTS

Dependent Variables	Groups	N	Mean	Std deviation
Understanding of Mathematical Concepts	Pre-test Control	32	45.17	8.501
	Post-test Control		56.17	9.190
	Pre-test Experimental	32	47.90	9.499
	Post-test Experimental		77.03	11.246
Higher-Order Thinking Skill (HOTS)	Pre-test Control	32	44.41	7.953
	Post-test Control		53.83	7.442
	Pre-test Experimental	32	46.86	7.946
	Post-test Experimental		76.81	9.345

Fig. 2 shows a diagram of the results of students' post-test understanding of mathematics concepts and HOTS by category. In the very high and high categories achieving learning completeness with a score of ≥75. These results show that the number of students who get a score of understanding mathematical concepts ≥75 reaches 44 students (69%), and 20 students (31%) obtained a value of <75. While the number of students who received a HOTS score ≥75 reached 42 students (66%), and 22 students (34%) obtained a score of <75. It shows better results compared to the results pre-test, which has not reached the specified competency score. Thus, the flipped classroom learning model meets the criteria of effectiveness based on the results of tests of understanding math concepts and HOTS.

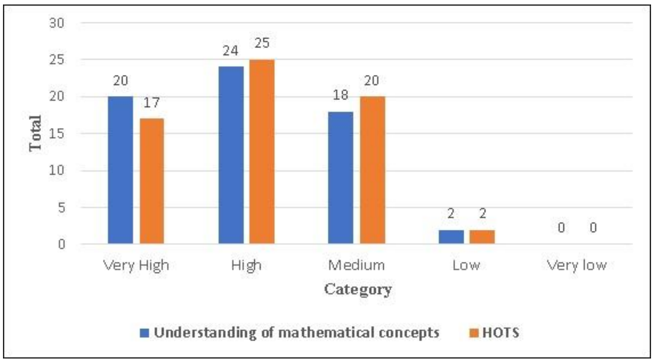

Fig. 2. Learning Test results.

Table VII shows a significant difference in average scores in understanding mathematical concepts (F = 132.017, p = 0,000) and higher-order thinking skills (F = 236.911, p = 0.000) between the experimental group and the control group. In addition, the existence of different treatments affected the average post-test scores of students in the experimental and control groups. Thus, the flipped classroom learning model is effective in increasing students' understanding of mathematical concepts and higher-order thinking skills compared to conventional models.

TABLE VII: THE RESULTS OF MANOVA

Dependent Variables	Type III Sum of Squares	df	F	Sig.
Understanding of Mathematical Concepts	13923.633	1	132.017	0.000
Higher-Order Thinking Skill (HOTS)	16905.008	1	236.911	0.000

This study successfully implemented the flipped classroom learning model to support junior high school student's ability to understand mathematical concepts and high-order thinking skills. The research statistics show that students in the experimental group are more superior than the control group. This shows that the flipped classroom learning model improves the understanding of mathematical concepts and higher-order thinking skills of the experimental group students. This learning model has provided an exciting learning experience for students. It's because the flipped classroom learning model includes several learning activities both outside the classroom and in collaborative classrooms through problem-solving activities using mathematical modeling, which impacts student performance in learning mathematics.

The ability to understand students' mathematical concepts increases in solving flat-sided geometrical problems. In the first indicator, students' understanding ability to determine the elements of a concept increases in the application of the flipped classroom learning model. Students can show illustrations and explain the elements of cube nets accurately and communicatively. Students can understand and visualize cube shapes through cube nets (See Fig. 3).

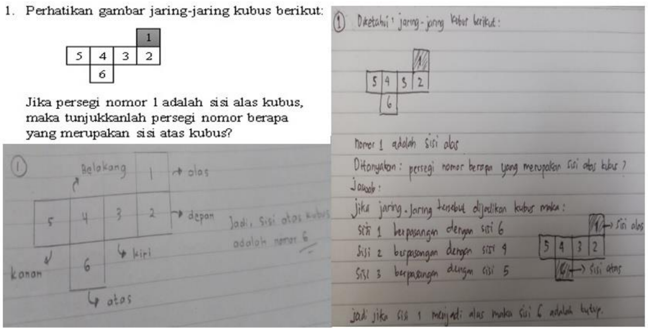

Fig. 3. Student response to concept understanding in indicator 1.

The second indicator increases the ability to recognize and associate relationships between concepts. Students can attribute and solve problems related to flat-side shapes in a precise, coherent, and communicative manner (See Fig. 4). Increased understanding of mathematical concepts in the experimental group proves that the application of the flipped classroom learning model can facilitate students' online learning by using videos to develop an understanding of mathematical concepts. This study aligns with the findings of Sari *et al.* [53] that the flipped classroom learning model using video positively affects students' understanding of mathematical concepts. In understanding activities, the flipped classroom learning model has given students responsibility for managing their learning and helping their thinking processes better. Students learn online by watching videos anywhere and anytime at their own pace. Students can watch the video repeatedly to better understand the concept of a flat-sided space and record essential points from the video. Students have a positive attitude towards using videos outside the classroom [54].

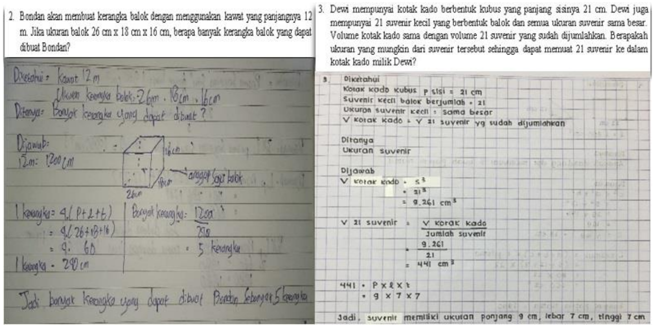

Fig. 4. Student response to concept understanding in indicator 2

Students gain learning experience through online discussion activities. For example, students are actively involved in finding the formula for the surface area of a cuboid by using their sentences communicatively and appropriately. This is done so that students can develop their thinking processes in understanding the concept of flat-sided shapes and increase student involvement in discussions. In Simplifying activities, students are also involved in completing pre-class quizzes. Student involvement greatly influences their success in achieving pre-class quizzes and their attitude toward the flipped classroom learning model [55]. In addition, this process can assist teachers in assessing students' comprehension abilities before entering class.

Increased understanding of the concept on the third indicator in the experimental group proves that applying the flipped classroom learning model can provide ideas for students to apply mathematical concepts in real life. Students can solve geometrical problems by justifying the correctness of information with communicative explanations and using appropriate procedures (See Fig. 5). This is supported by learning activities in collaborative classes using mathematical modeling to apply the flipped classroom learning model. This study aligns with the findings of Sokolowski [47], who claim that mathematical modeling can help students understand and apply mathematical concepts. In mathematizing activities, the flipped classroom learning model allows students to remember and understand mathematical concepts better because the teacher clarifies essential concepts in the video.

In working mathematical activities, the flipped classroom learning model helps students actively participate in groups and work together to solve real problems with teacher guidance. Working with peers can help other students build their confidence and increase their understanding of mathematical concepts [37]. This collaborative activity supports thinking and reasoning processes so that students can be involved in learning and improve their understanding of the concept. Collaboration is essential in completing tasks related to hands-on and project-based learning activities. This finding aligns with the results of Dori and Kohen *et al.* [56] that applying the flipped classroom learning model can improve students' conceptual understanding in solving problems and completing projects. The existence of teamwork in collaborative classes can enhance active learning in class to achieve more meaningful learning outcomes.

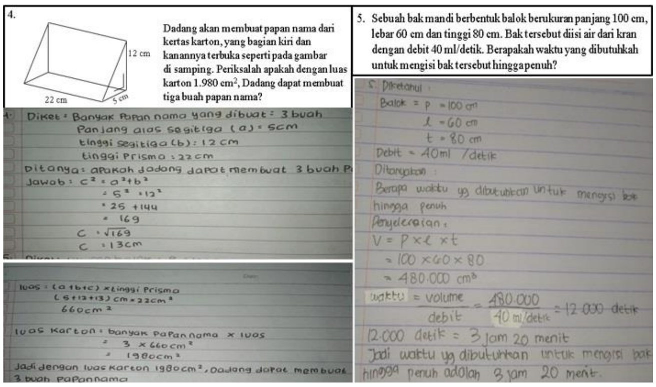

Fig. 5. Student response to concept understanding in indicator 3.

Research findings revealed that the application of the flipped classroom model in supporting students' understanding of mathematical concepts could significantly affect student achievement and motivation compared to conventional teaching [57]. Therefore, it is conclusive that students who have a good understanding of mathematical concepts can influence their learning achievement so that they can improve their learning performance. This study also shows that the flipped classroom learning model is more beneficial for students with lower achievements than those with high achievers and moderate achievers in understanding mathematical concepts. Because low-achieving students get more attention from the teacher, they discuss problems to understand mathematical concepts. Wei *et al.* [58] explained that this learning model has helped students think deeply both in class and outside the classroom and can interact well with learning videos, thereby increasing their learning achievement.

The findings of this study also reveal that the flipped classroom learning model effectively increases HOTS. HOTS capabilities include the ability to analyze, evaluate and create. The application of the flipped classroom learning model can improve students' analytical skills on the topic of flat-sided shapes (See Fig. 6). Increasing the HOTS of students in the experimental group revealed that the application of the flipped classroom learning model had provided a way for students to use analytical skills, namely being able to differentiate, compare and solve problems of flat-sided shapes.

Working mathematically through problem-solving activities in completing group assignments helps students understand the topic. In this way, students can interact with their group mates to give opinions, and skilled students can help other students avoid misunderstandings. So that at the analysis level, students can apply concepts and solve mathematical problems correctly in the tests given. This finding aligns with the results of Clark [37] that the flipped classroom learning model can increase engagement and communication compared to traditional learning. This learning model strongly supports interaction between students with peers, and teachers.

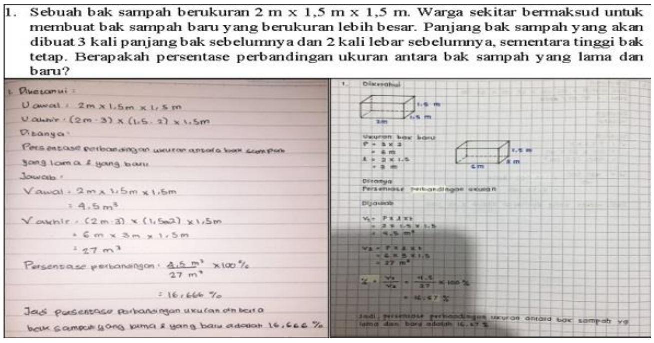

Fig. 6. Student responses to ability of analysis.

The findings of this study indicate that the application of the flipped classroom learning model can improve students' evaluation abilities (See Fig. 7). The increase in student HOTS in the experimental group revealed that the application of the flipped classroom learning model had provided space for students to examine and criticize the correctness of information from real problems related to flat-sided space shapes. Students can apply mathematical concepts in everyday life and check the truth of the information. This model can help other groups to evaluate information from these real problems. Giving group assignments in the form of modeling assignments can improve mathematics learning. The findings of Riyanto and Zulkardi *et al.* [59] show that the mathematical modeling approach effectively improves the quality of learning mathematics. This learning uses students' thought processes in completing modeling assignments so that students can be involved in learning mathematics.

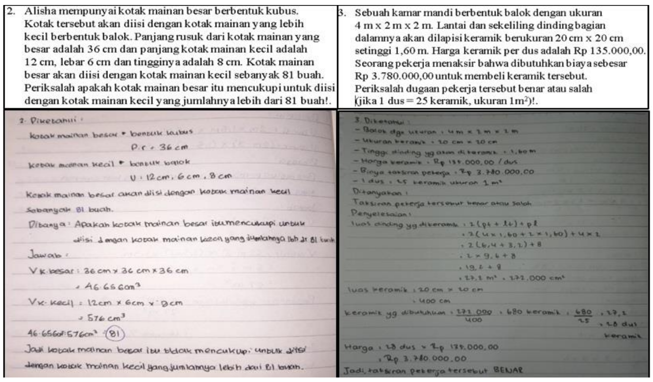

Fig. 7. Student responses to ability of evaluation.

Application of the flipped classroom learning model can improve creative abilities (see Fig. 8). The increase in student HOTS in the experimental group shows that the application of the flipped classroom learning model provides a way for students to develop creative abilities, namely formulating, designing, and producing. This flipped classroom learning model has the advantage of connecting thinking skills with real problems through mathematical modeling. Problem solving activities in collaborative classes use mathematical modeling activities. Sokolowski [47] found that mathematical modeling activities have a positive learning effect compared to traditional methods. Furthermore, the findings of Tezer and Cumhur [46] show that mathematical modeling can improve students' mathematics achievement. Thus, the flipped classroom learning model can potentially enhance problem-solving skills.

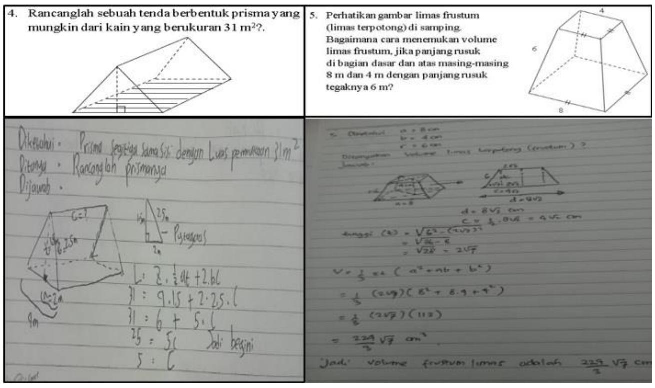

Fig. 8. Student responses to ability of creation.

Improving the ability of analysis, evaluation, and creation proves that applying the flipped classroom learning model can increase students' HOTS. The research is in line with Palinussa *et al.* [60] that the flipped classroom learning model can improve HOTS. Students' thinking skills have increased at the level of analysis, evaluation, and creation, and student test results show that three out of 36 students are in the very high category, and no students are in the deficient category. In addition, the flipped classroom learning model presents real-world problems that can be solved using mathematical modeling. This aligns with the findings of Samo *et al.* [61] that learning by giving contextual problems or real problems can increase students' HOTS and produce higher-order thinking skills in the high category. These results indicate that the flipped classroom learning model can increase student HOTS by integrating mathematical modeling.

IV. CONCLUSION

This study aims to determine the effectiveness of the flipped classroom learning model compared to conventional teaching methods. Interventions were carried out in both approaches by providing the same learning materials. The results showed that students' ability to understand mathematical concepts and higher-order thinking skills delivered using the flipped classroom learning model differed significantly from those who were taught using conventional teaching method. The flipped classroom learning model has succeeded in increasing understanding of mathematical concepts and higher-order thinking skills of junior high school students. The flipped classroom learning model can support students' problem-solving abilities and increase their collaboration in learning mathematics. The flipped classroom is an active learning model involving students in learning and making mathematics teaching more meaningful. Other researchers can carry out further research by considering different variables. Mathematics teachers can also consider these findings or use learning models or other strategies that support mathematical modeling-based learning to improve students' understanding of mathematical concepts and higher-order thinking skills.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Diana determined research topics, analyzed data, developed test instruments, interpreted data, and wrote an initial paper. Herman Dwi Surjono provided advice regarding research and development and reviewed the analytical framework, analysis, and discussion. Ali Mahmudi provided advice on research and development, reviewed analysis and discussion, and validated test instruments and research rubrics. All authors had approved the final version.

REFERENCES

[1] K. Ratheeswari, "Information communication technology in education," *Journal of Applied and Advanced Research*, vol. 3, no. 1, pp. 45−47, May 2018.

[2] D. S. Nahdi, "Keterampilan matematika di abad 21," *Jurnal Cakrawala Pendas*, vol. 5, no. 2, pp. 133–140, Jul 2019.

[3] D. Bourn, *Understanding Global Skills for 21st Century Professions*, Palgrave: Macmillan, 2018.

[4] Ministry of Education and Culture, *Permendikbud Number 24 of 2016 Concerning Core and Basic Competencies in Primary and Secondary Education Curriculum 2013*, Jakarta, Indonesia: Ministry of Education and Culture of the Republic of Indonesia, 2016.

[5] Ministry of Education and Culture, *Permendikbud Number 22 of 2016 Concerning the Objectives of Learning Mathematics in Primary and Secondary Education Curriculum 2013*, Jakarta, Indonesia: Ministry of Education and Culture of the Republic of Indonesia, 2016.

[6] E. Hoosain, "What does it mean to understand mathematics?" *Humanistic Mathematics Network Journal*, vol. 9, no. 25, pp. 20–22, 2001.

[7] National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics*, Reston, VA: NCTM,1989.

[8] National Council of Teachers of Mathematics, *Principles to Action: Ensuring Mathematical Success for All*, Reston, VA: NCTM, 2014.

[9] National Council of Teachers of Mathematics, *Principles and Standards for Schools Mathematics,* Reston, VA: NCTM, 2000.

[10] S. Brookhart, *How to Assess Higer-Order Thinking Skills in Your Classroom*, ASCD Alexandria, Virginia USA, 2010.

[11] S. Jailani, H. Retnawati, Bukhori, E. Apino, H. Djidu, and Z. Arifin, *Desain Pembelajaran Matematika untuk Melatihkan Higher Order Thinking Skill*, Yogyakarta: UNY Press, 2018.

[12] S. Pogrow, "HOTS revisited: A thinking development approach to reducing the learning Gap After Grade 3," *Phi Delta Kappan,* vol. 87, no. 1, pp. 64–75, Sep 2005.

[13] Puspendik, *Mastery of 2018/2019 National Exam Material*, Jakarta, Indonesia: Center for Educational Assessment of the Ministry of Education and Culture of the Republic of Indonesia, 2019.

[14] H. Retnawati, "Diagnosing the junior high school student's difficulties in learning mathematics," *International Journal on New Trends in Education and Their Implications,* vol.8, no. 1, pp. 33–50, Jan 2017.

[15] E. S. Indrayany and F. Lestari, "Analysis of junior high school students' difficulties in solving geometric problems and factors causing student difficulties in terms of van Hiele's theory," *Journal of Math Educator Nusantara: A Forum for Publication of Scientific Writing in the Field of Mathematics Education,* vol 5 , no 2, pp 109–123, Nov 2019. (in Indonesian)

[16] Junitasari and F. Hayati, "Analysis of the ability to understand mathematical concepts of junior high school students on blocks and cubes material," *Mathematixs Educational Journal,* vol. *2* no. 1, pp. 14–25, Apr 2019. (in Indonesian)

[17] H. Retnawati, J. Arlinwibowo, and E. Sulistyaningsih, "The Students difficulties in completing geometry items of national examination,"

International Journal on New Trends in Education and Their Implications, vol 8 no. 4, pp. 28–41, Okt 2017.

[18] D. N. Utami, B. Kusmanto, and S. A. Widodo, "Error analysis in working on geometry problems," *Journal of Mathematics and Science Education*, vol. 7 no. 1, pp. 37–44, Mar 2019. (in Indonesian)

[19] S. Aminah and A. Rohayati, "Analysis of high-level thinking skills of junior high school students in solving HOTS geometry problems based on learning achievement," *Journal of Research and Scientific Work,* vol. 21, no. 1, pp. 141–151, Jul 2021.

[20] Meryansumeka, Zulkardi, R. I. I. Putri, and C. Hiltrimartin, "Kesulitan siswa dalam menyelesaikan permasalahan geometri level higher order thinking skill," Supremum *Journal of Mathematics Education,* vol 5, no. 2, pp. 189–198, Jul 2021.

[21] C. P. Tanudjaya and M. Doorman, "Examining higher-order thinking in Indonesian lower secondary mathematics classrooms," *Journal on Mathematics Education,* vol. 11, no. 2, pp. 277–300, May 2020. (in Indonesian)

[22] M. Selan, F. Daniel, and U. Babys, "Analysis of students' mathematical literacy skills in solving PISA questions about change and relationship content," *Journal of Mathematics and Mathematics Education,* vol. 11, no. 2, pp. 335–345, Dec 2000.

[23] M. M. Simalango, Darmawijoyo, and N. Aisyah, "Students' difficulties in solving PISA questions on content change and relationship levels 4, 5 and 6 at SMPN 1 Indralaya," *Journal of Mathematics Education,* vol. 12, no. 1, pp. 43–58, Apr 2018. (in Indonesian)

[24] P. Febrianti and Nurjanah, "Difficulties of junior high school students in solving PISA 2021 questions," *Transformation: Journal of Mathematics and Mathematics Education*, vol. 6, no, 1, pp. 13–24, May 2022. (in Indonesian)

[25] E. D. Wulandari, "The analysis of external factors that influence learning difficulty in learning mathematics for grade 4 students of SD Negeri 125 Pekanbaru," *Jurnal PAJAR,* vol. 4, no. 4, pp. 738–744, Jul 2020.

[26] B. Murtiyasa, S. Rejeki, and N. Ishartono, "Profile of students' error in solving mathematics word problems based on PISA frameworks," in *Proc. SEMANTIK Conference of Mathematics Education (SEMANTIK 2019),* Atlantis Press, Aug 2020, pp. 135-137.

[27] B. Tanujaya, R. C. I. Prahmana, and J. Mumu, "Mathematics instruction to promote mathematics higher order thinking skills of students in Indonesia: Moving forward," *TEM Journal*, vol. 10 no. 4, pp. 1945–1954, Nov 2021.

[28] K. Y. Lee and Y. C. Lai, "Facilitating higher-order thinking with the flipped classroom model: A student teacher's experience in a Hong Kong secondary school," *Research and Practice in Technology Enhanced Learning,* vol. 12 no. 1, pp. 1–14, Dec 2017.

[29] S. Aslan, "Teacher candidates' experiences with the flipped classroom model: A phenomenological approach," *International Journal of Contemporary Educational Research*, vol. 7, no. 2, pp. 202–211, Dec 2020.

[30] P. Leszczyński, A. Charuta, B. Łaziuk, R. Gałązkowski, A. Wejnarski, M. Roszak, and B. Kołodziejczak, "Multimedia and interactivity in distance learning of resuscitation guidelines: A randomised controlled trial," *Interactive Learning Environments,* Feb 2018.

[31] J. Bergmann and A. Sams, "Flip your classroom: Reach every student in every class," *International Society for Technology in Education,* Jun 2012.

[32] J. W. Baker, "The classroom flip: Using web course management tools to become the guide by the side," in *Proc. 11th international Conference on College Teaching and Learning,* Jacksonville, Florida Community College, 2000.

[33] J. L. Bishop and M. A. Verleger,"The flipped classroom: A survey of the research," *ASEE Annual Conference and Exposition, Conference Proceedings,* pp. 23–1200, Jun 2013.

[34] Z. Zainuddin and S. H. Halili," Flipped classroom research and trends from different fields of study," *International Review of Research in Open and Distributed Learning,* vol. 17, no. 3, pp. 313–340, Apr 2016.

[35] T. Muir, "Self-determination theory and the flipped classroom: A case study of a senior secondary mathematics class," *Mathematics Education Research Journal*, vol. 33, pp. 569–587. Sep 2020.

[36] M. Jdaitawi, "The effect of flipped classroom strategy on students learning outcomes," *International Journal of Instruction*, vol. 12, no. 3, pp. 665–680. Jul 2019.

[37] K. Clark, "The effects of the flipped model of instruction on student engagement and performance in the secondary mathematics classroom," *The Journal of Educators Online,* vol. 12, no. 1, pp. 91–115, Jan 2015.

[38] K. Fulton, "Upside down and inside out : Flip your classroom to improve student learning," *Learning & Leading with Technology*,vol. 39, no. 8, pp. 12–17, 2012.

[39] C. K. Lo and H. F. Hew, "A comparison of flipped learning with gamification, traditional learning, and online independent study: The effects on students' mathematics achievement and cognitive engagement," *Interactive Learning Environments,* vol. 28, no. 4, pp. 464–481, May 2020.

[40] E. Apriska and Sugiman, "Flipped classroom research trends in mathematics learning in Indonesia," *Journal of Physics: Conference Series,* vol. 1613, no. 1, 012030, IOP Publishing, Aug 2020.

[41] N. B. Milman, "The flipped classroom strategy: What is it and how can it best be used?" *Distance Learning*, vol. 9, no. 3, p. 85, Jul 2012.

[42] M. K. Kim, S. M. Kim, and J. Getman, "The experience of three flipped classrooms in an urban university: An exploration of design principles," *The Internet and Higher Education,* vol. 22, pp. 37–50, July 2014.

[43] B. Tanujaya, J. Mumu, and G. Margono, "The relationship between higher order thinking skills and academic performance of student in mathematics instruction," *International Education Studies*, vol. 10, no. 11, pp. 78–85, 2017.

[44] W. Blum and D. Leiß, "Deal with modelling problems," in *Proc. Mathematical Modelling: Education, Engineering and Economics-ICTMA*, vol. 22, Aug 2007, pp. 222–231.

[45] W. Blum, "Teaching and learning mathematics in context," *Mathematical Modelling in Mathematics Education and Instruction*, vol. 1, pp. 3–14, 1993.

[46] M. Tezer and M. Cumhur, "Mathematics through the 5E instructional model and mathematical modelling: The geometrical objects," *Eurasia Journal of Mathematics, Science and Technology Education*, vol. 13, no. 8, pp. 4789–4804, Jul 2017.

[47] A. Sokolowski, "The effects of mathematical modelling on students' achievement-meta-analysis of Research," *IAFOR Journal of Education*," vol. 3, no. 1, pp. 93–114, 2015.

[48] M. L. K. P. Santos, R. R. Belecina, and R. V. Diaz, "Mathematical modeling: effects on problem solving performance and math anxiety of students," *International Letters of Social and Humanistic Sciences,* vol. 65, pp. 103–115, 2015.

[49] K. Maaß, "Classification scheme for modelling tasks," *Journal Mathematik-Didaktik,* vol. 31, no. 2, pp. 285–311, Oct 2010.

[50] B. Riyanto and R. I. I. Putri, "HOTS on mathematical modelling approach in primary school," *Journal of Physics: Conference Series*. vol. 1188, no. 1, 012088, IOP Publishing, Mar 2019.

[51] W. R. Shadish, T. D. Cook, and D. T. Campbell, *Experimental and Quasi-Experimental Designs for Generalized Causal Inference*, pp. 103–134, Boston MA: Houghton Mifflin, 2002.

[52] L. W. Anderson and D. Krathwohl, *A Taxonomy for Learning, Teaching and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives*, New York: Longman, 2001.

[53] L. P. Sari, M. Handika, E. Rosita, M. Sari, B. S. Anggoro, and F. G. Putra, "The flipped classroom strategy using learning video: Applied toward the ability to understand mathematical concept," *Journal of Physics: Conference Series*, vol. 1155, no. 1, 012088, IOP Publishing, Feb 2019.

[54] T. Long, J. Logan, and M. Waugh, "Students' perceptions of the value of using videos as a pre-class learning experience in the flipped classroom," *TechTrends*, vol. 60, pp. 245–252, 2016.

[55] M. Cevikbas and G. Kaiser, "Student engagement in a flipped secondary mathematics classroom," *International Journal of Science and Mathematics Education,* pp. 1–26, 2021.

[56] Y. J. Dori, Z. Kohen, and B. Rizowy, "Mathematic for computer science: A flipped classroom with optional project," *EURASIA Journal of Mathematics, Science and Technology Education*, vol. 16, no. 12, pp. 1–20, 2020.

[57] K. K. Bhagat, C.-N.Chang, and C.-Y. Chang, "The impact of the flipped classroom on mathematics concept learning in high school," *Journal of Educational Technology & Society,* vol 19, no. 3, pp. 134–142, Jul 2016.

[58] X. Wei, I. L. Cheng, N. S. Chen, X. Yang, Y. Liu, Y. Dong, X. Zhai, and Kinshuk, "Effect of the flipped classroom on the mathematics performance of middle school students," *Educational Technology Research and Development,* vol. 68, no. 3, pp. 1461–1484, 2020, https://doi.org/10.1007/s11423-020-09752-x

[59] B. Riyanto, Zulkardi, R. I. I. Putri, and Darmawijoyo, "Senior high school mathematics learning through mathematics modeling approach," *Journal on Mathematics Education*, vol. 10, no. 3, pp. 425–444, Sep 2019.

[60] A. L. Palinussa, C. Laamena, and M. B. Mananggel, “Development of learning models flipped classroom to increase the mathematical higher order thinking skills (HOTS) of students ın high school,” *Technology Reports of Kansai University*, vol. 63, no. 3, pp. 7349–7355, Mar 2021.
[61] D. D. Samo, D. Darhim, and B. Kartasasmita, “Developing contextual mathematical thinking learning model to enhance higher-order thinking ability for middle school students,” *International Education Studies,* vol. 10, no.12, pp. 17–29, 2017.

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