Evaluating Students' Perception of Visual Mathematics in Secondary Geometry Education: A Mixed Methods Investigation

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***Abstract*—The present study aimed to assess the perspectives of female secondary school students on the integration of visual mathematics in engineering lessons. Employing a mixed-methods approach, specifically a convergent parallel design, qualitative data were gathered through observations and interviews (both individual and focus groups), while quantitative data were collected using a questionnaire. Video clips of engineering unit lessons were utilized as educational materials. The study included 135 female students in the first year of secondary school, with 74 participating in qualitative segments, divided into focus groups and individual interviews. SPSS was employed for quantitative data analysis, calculating means and standard deviations, while thematic analysis was applied for qualitative data. Key findings indicate that visual displays significantly contributed to enhancing female students' comprehension of mathematical concepts, facilitating the understanding of inter-concept relationships and their application in engineering. The incorporation of visual aids also positively impacted the mathematical coherence abilities of female students, establishing connections between mathematics, other subjects, and real-life situations. Furthermore, visual presentations in engineering topics were found to improve female students' attitudes toward learning mathematics, fostering increased self-confidence and enjoyment in learning various engineering subjects. Recommendations from the study include the need for training both male and female teachers in designing effective visual presentations for teaching mathematical concepts.**

***Keywords*—visual mathematics, mathematical representation, mixed study, secondary-school stage, geometry**

I. Introduction

Mathematics occupies a prominent place in school curricula for many reasons. First, it helps the individual to understand social and natural phenomena in quantitative terms. Second, the field of teaching mathematics represents an essential pillar in developing students' various abilities when exploring and investigating mathematical knowledge throughout academic stages. Third, one of these abilities is sight, which allows a person to interact with the outside world. Through the sense of sight, a person receives a great deal of knowledge, as some studies that dealt with the human brain showed that more than 80 % of knowledge is received by the eye [1].

Investing the visual abilities of students to improve and raise their performance in mathematics requires taking care of the methods of presenting mathematical knowledge, and seeking to present mathematics in a manner that reduces abstraction and brings it closer to the mind of the learner. Many scholars indicated that mathematics is the "science of patterns," and that learners should be provided with opportunities to study the mathematical structures that can be applied to other contexts which may themselves be a subject for further investigation [2].

On the other hand, the National Council of Teachers of Mathematics in the United States of America [3] indicated that representations are the only way to record and analyze these patterns, and that representation is an effective tool to solve the difficulties of understanding mathematics; students should be encouraged to represent their mathematical ideas as representation is a tool that facilitates learning mathematics and communication with others about mathematical fields. The Council added students' personal representations help them investigate mathematical ideas and solve problems. Goldin [2] indicated that visual representations play an auxiliary role in expressing mental representations, as they are a language expressed through visual images that allow the transfer of the concept and idea. On the other hand, the study of National Council of Teachers of [3] showed many problems that accompany students' learning of mathematics, including abstract phenomena and concepts and poor understanding of the concepts they study.

The National Council of Teachers of [3] highlighted the role of computers in helping the learner expand representations. In this context, the Council emphasized the principle of technology as a principle of school mathematics, and that electronic technologies are essential tools in teaching, learning, mathematical operations, and the provision of visual images of mathematics, due to technology's potentials in terms of graphic power, the provision of strong visual models, and the provision of multiple perceptions to see mathematical ideas. It adds more details to students' learning and gives them more room to expand their mathematical knowledge, which makes the widespread use of technology necessary to enrich students' learning of mathematics [4].

A number of studies dealt with the visual representation of mathematical knowledge, which led to the emergence of the term visual mathematics. Duran & Bekdemir [5] showed that there is no unified definition of visual mathematics, and they defined it as a computer-based secondary program according to the standards of the National Council for Teachers of

Mathematics (NCTM) [3]. Geçici & Türnüklü [6] defined visual mathematics as visual or tangible production such as diagrams, number lines, graphs, arrangements of tangible objects, processing, physical models, mathematical expressions, formulas, equations, or graphics on a computer screen or calculator. The research team defines visual mathematics as a method of presenting mathematical content by representing mathematical (conceptual and procedural) knowledge in a visual representation instead of a written image.

The reason for the absence of a unified definition of visual mathematics may be due to its being a recent trend, in addition to the overlap in the sections and classifications of mathematical representations. Geçici & Türnüklü [6] indicated that the educational literature dealt with representation in different ways, and this appears in the Gilbert's [7] classification of representation into two categories: the symbolic and the visual. Symbolic representations express abstract structures such as signs, discourses, or symbols, while visual representations express structures that emphasize the visual form of information such as pictures, charts, or graphics. Gilberts [7] pointed out that the visual representation of a mathematical problem can be internal or external: the internal visual representation is an object of the mind created from our experiences; an external visual representation includes paper and pencil drawings, models or other physical manipulations, and technological visualizations of a problem.

Visual mathematics has a theoretical foundation in the field of cognitive psychology and learning research, exemplified by the ideas of Piaget and Bruner and the theory of learning based on brain research. One of Piaget's ideas about cognitive growth is that it is the product of the individual's direct interaction with the surrounding environment. In addition, he classifies the stages of cognitive development in children according to their age. The stage of tangible processes extending from (7–12) years represent basic education in the primary school. In this stage, the individual begins to think in a sensory logical way, not in an abstract logical one. This thinking depends on dealing with objects and sensible things [8]. The learner interacts, in the educational environment, with visual mathematics as one of the sources of learning. In addition, visual mathematics presents and represents abstract mathematical knowledge in different visual forms that suit learners' levels of thinking.

The process of representation represents plays a focal role in cognitive growth according to Bruner's theory [9]. Burner identified three methods that the individual uses during learning: sensory representation, visual representation, and symbolic representation [10]. Visual mathematics can be considered a method for representing mathematical ideas through the first and second stages: sensory representation through direct sensory dealing with objects, and the symbolic representation of knowledge through images and drawings.

As for the learning theory based on brain research, it supports visual mathematics with its vision of how the brain works during learning. It sees that the structure of the brain is divided into two hemispheres: the left hemisphere and the right hemisphere, and that the human ability to learn increases depending on the use of these two hemispheres together. Each hemisphere of the brain has different functions: the left hemisphere of the brain is responsible for the analytical aspects represented in words and symbols, while the right hemisphere is concerned with structural aspects such as drawings, shapes, images and models. As the brain works in a complete and unified manner, its two hemispheres overlap. Therefore, it is necessary to focus in the educational environment on both hemispheres of the brain to activate learning for learners [11].

This supports the need to use visual mathematics as a source of learning in the teaching process to clarify the mathematical idea. Linking visual representations and mathematical ideas helps to develop brain connections by seeing mathematics in different numerical, visual, algebraic and verbal ways. A teacher's deep content knowledge, technical knowledge, and knowledge of learner characteristics are prerequisites for the successful employment of visual mathematics in the mathematics classroom. The preparation of visual presentations of mathematical knowledge may go through several stages, namely: reading the mathematical content in depth, defining the basic mathematical knowledge in the lesson, linking knowledge to the environment surrounding the student, and preparing and designing appropriate mathematical experiences to represent basic mathematical knowledge.

Visible mathematics contributes by presenting various and different visual representations of mathematical concepts and procedures by providing effective teaching of mathematical knowledge in ways that suit many students. Joo [12] indicated the role visual mathematics plays in helping students conceptualize mathematical topics. Moreover, the visual representation of mathematical concepts enhances students' understanding and confidence in their own learning, and supports their independence when learning math topics both inside and outside the classroom when learning individually. In addition, visual mathematics contributes to the development of good thinking, solving visual mathematical problems, keeping representations in the mind, and giving importance to mathematics [5]. Furthermore, visual mathematics helps give students access to deep and new concepts and raise the level of mathematical achievement [13]. Visual mathematics can help students get acquainted with advanced mathematical concepts [14]. This may justify the findings of the study Abdul-Rahman [15] regarding the attention that the mathematics textbook in Japan pay to the visual language in clarifying mathematical concepts and procedures. In addition, visual presentations of mathematical knowledge improve teaching environments and communication between teachers and students [16].

Several studies dealt with visual mathematics, including [17], which aimed to examine the mathematical perceptions of mathematics teachers and reached results, including that there are significant differences in the mathematical visual perceptions of mathematics teachers according to the grade level and the educational status of parents. Lubis *et al*. [18] aimed to reveal the perceptions of students and teachers about educational videos in mathematics and found out that the majority of students and teachers have positive perceptions about educational videos in terms of helping them remember information and understand more about the mathematical concept. Ev Cimen & Aygüner [19] aimed to analyze the relationship between eighth-grade students' perceptions of self-efficacy of for visual mathematics literacy and their actual performance and found out that there is no correlation between the scores of the perception scale and the results of the actual performance test. When examining this discrepancy through interviews, the researchers concluded that the students were not well aware of their own self-efficacy. As for the study of Florensia & Suryadibrata [20], it found visual presentation increases students' interest in learning mathematics and their involvement in learning it. Florensia & Suryadibrata [20] found out that visual mathematics plays a role in improving students' algebraic expression. This study revealed a positive correlation between students' intelligence, visual perception skills, and mathematical achievement. Garzon & Casinillo [21] aimed to study the differences between two groups, one of which studied in the traditional method and the other used visual representation, and their study found out that there were differences in favor of the experimental group in terms of their ability to solve mathematical problems.

From the foregoing, it is evident that visual mathematics plays a manifest role in supporting students' learning of mathematics through their visual perception capabilities, and it contributes to the embodiment of mathematical knowledge, deepening their understanding of concepts, procedures and mathematical proofs and linking them to applications in life, which makes students feel pleasure and increases their motivation and curiosity to learn mathematics and benefit from its tools in daily life, as mathematics is a science that contains realistic visual applications that should be reflected in school mathematics education.

A. Study Problem

The study addresses a significant gap identified through various sources. The problem emerges from the diverse learning styles of students, influenced by differences in abilities and academic readiness. Visual perception variations among students, highlighted by Refaat [22] and the positive correlation between academic achievement in mathematics and visual working memory, as revealed by Salem and Muhammad [23], underscore the need for attention to individual differences. Abdul-Rahman [15] emphasizes the use of visual language in explaining mathematical knowledge, while Al-Moatham [24] suggests exploring effective visual organizers in mathematics teaching. The recommendations from [3, 13, 21, 25] collectively emphasize the importance of integrating visual mathematics into teaching strategies.

However, the research team observed a reluctance among mathematics teachers to employ visual representations, focusing predominantly on procedural knowledge at the secondary school level. This observation further reinforces the identified gap. To address this issue, the study proposes the application of visual mathematics in mathematics lessons. The intervention aims to not only bridge the observed gap in teaching methodologies but also to unveil the diverse opinions of learners regarding the integration of visual mathematics. By implementing visual mathematics strategies, the research seeks to enhance the effectiveness of teaching practices and better cater to the diverse learning styles of students, aligning with the recommendations from the aforementioned sources.

B. Study Objectives

The study aimed to identify female students' opinions of employing visual mathematics in geometry lessons at the secondary school.

C. Study Questions

The study sought to answer the following question: What are female students' opinions of employing visual mathematics in teaching geometry topics at the secondary school?

D. Importance of the study

The current study derives its importance from the fact that it dealt with one of the recent trends in teaching mathematics, i.e., visual mathematics which represents an addition to the field of mathematics education. The current study surveys female students' opinions of teaching mathematics by means of visual presentation and the factors influencing it, which in turn may contribute to providing those in charge of developing mathematics education and decision-makers with ideas that should be taken into account when developing school mathematics curricula, mathematics teacher preparation programs and professional development programs. It is also hoped that the study may direct researchers in the field of mathematics teaching to deal with visual mathematics from different angles and different academic stages.

II. Methodology

A. Sample

The study employed a mixed-methods approach, engaging a total of 135 female students in active participation. Of these, 121 students responded to a quantitative questionnaire, providing valuable insights for the quantitative aspect of the research. Additionally, 74 students participated in the qualitative segment through group interviews, while six students took part in individual interviews, representing two students from each major class within three randomly chosen first-year secondary school classes (Table 1).

Table 1. Data of the participants in the study's individual interviews

Participant's code	**Participant's description**	
	Academic level	**Extracurricular activities related to mathematics**
S1	Above average	participated in making a conceptual map on mathematics
S2	Average	participated in making a PowerPoint presentation on mathematics
S3	Above average	participated in making an infographic on mathematics
S4	Average	participated in the making a brochure on mathematics
S5	Above average	participated in a Canva presentation on mathematics
S6	Above average	participated in making a visual video on mathematics

These classes, selected from a pool of six, collectively formed the primary respondent sample for the study. The integration of both quantitative and qualitative data collection methods, coupled with the strategic selection of classes and participants, aimed to offer a comprehensive and multifaceted understanding of the study's objectives.

Following a mixed-methods design, specifically a convergent parallel design as defined by Creswell [26], the study collected and integrated both quantitative and qualitative data through distinct research approaches. This approach allowed for a more profound, comprehensive, and credible analysis of the study problem, focusing on exploring the perspectives of female secondary school students regarding the integration of visual mathematics in engineering lessons.

The analysis of quantitative data involved SPSS for means and standard deviations, while thematic analysis was applied to the qualitative data. The subsequent Fig. 1 illustrates the steps of data collection in this comprehensive and well-structured study.

Qualitative data were obtained through observations and interviews, including both individual and focus group

sessions. On the other hand, quantitative data were collected using a questionnaire, with educational materials such as video clips of engineering unit lessons provided. A total of 135 first-year female secondary school students participated, with 74 engaging in qualitative segments through focus groups and individual interviews.

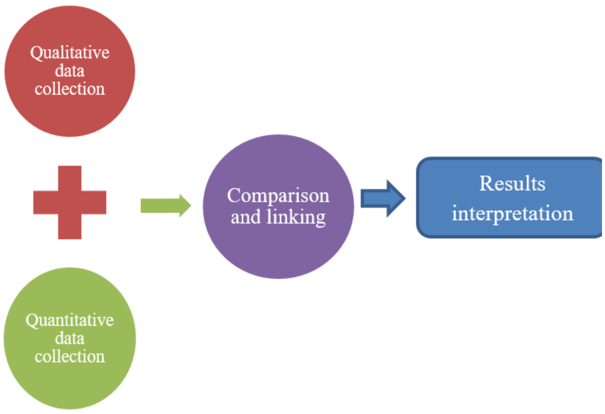

Fig. 1. Illustration of the quantitative and qualitative data collection steps.

B. *Explanation of the Steps to Study Implementation*

The first stage: The researchers Following the meticulous creation and review of the instructional videos by the research team, each video was strategically introduced at the beginning of every class session. This approach aimed to stimulate student engagement through individual reflection, followed by group discussions on the prominent concepts and skills covered. The students demonstrated a positive response, actively applying their understanding of mathematical concepts acquired through video viewing. Remarkably, the videos facilitated a rapid mental grasp of quadrilateral properties through visual contemplation.

The instructional strategy extended across three classes, each consisting of 45 female students. Aligned with the study's focus on the "Geometrical Shapes" unit, comprising five lessons within a three-week period and a total of 15 lessons, the videos were thoughtfully tailored to correspond with the specific content and duration of this instructional segment. This strategic alignment aimed to enhance the effectiveness of the intervention and promote a comprehensive understanding of geometrical concepts among the students.

The second stage: The research team used a set of quantitative and qualitative tools to collect data, as follows:

- Free classroom observation, to observe the role of visual mathematics on female students' performance and their interaction during the presentation of visual mathematics in mathematics lessons; a number of observations were made.
- An open interview (individual interviews and focus group) to identify female students' opinions of visual mathematics, which took place in two stages during the implementation of the study and after its completion.
- A quantitative questionnaire to reveal female students' opinions of visual mathematics, which was applied after completing the teaching of the topics of the geometry unit that was taught with visual presentations (visual mathematics).

The third stage: comparison and linking: In this stage, the research team compared the results of the quantitative and qualitative study individually. After that, the team members held group discussions.

C. *Study Population*

The study population consisted of all female students in the first secondary-school grade at Sondos Bint Khalid School in the academic year 2023. The number of the study population was 277 female students.

D. *Study Tools*

Study materials: The study used educational video clips as a study material for the topic of "Quadrilaterals" in the fifth unit of the first secondary-school grade. This chapter was chosen because it provides an opportunity for employing visual mathematics, simplifying mathematical concepts, developing visual thinking skills, and understanding mathematical geometry in a fun and attractive way. The content of the this chapter includes all the knowledge, skills, and experiences that the student is expected to acquire after watching the educational videos. The educational video clips were produced through several stages:

1) **Visual educational content**: The research team analyzed the content of the topic (Quadrilaterals) in order to identify the mathematical concepts and procedures included in the unit lessons, and then produce the video clips in the light of the results of the analysis. The educational video clips covered 5 main topics of the mathematics lessons of the first secondary-school grade textbook. The following is an explanation of these topics, concepts, basic skills, and mathematical theories included in the videos:
2) **Production of educational video clips**: Educational video clips related to the objectives of the lesson and the mathematical concepts and procedures included in it were produced, taking into account that each video clip is a source of mathematical knowledge that serves a specific goal in the lesson, as well as that the duration of the video clip does not exceed 2 minutes to be of a small size so that it can be easily circulated among the female students.

Moreover, the titles of these clips were numbered in order to maintain their sequence. These videos have also been coded to link them to assessments and tasks related to the same goal. The research team used a number of computer programs to design the video clips, such as PowerPoint and Class Point.

3) **Showing the educational video clips to the reviewers**: After completing the preparation of the educational videos, they were shown to a group of 3 reviewers specialized in teaching mathematics at the secondary school level for the purpose of getting their observations and suggestions in terms of the videos' objectives, content, and interactive visual techniques and their colors and method of presentation, as well as the validity of the visual presentations and their suitability for the characteristics of the secondary school stage. Amendments were made in the light of their feedback until the videos were ready for application for a period of two weeks (14 days), at a rate of 5 classes per week. Thus, the visual clips took their final form.

E. Procedures

Because the study used the mixed method, a quantitative tool, i.e., a questionnaire, and qualitative tools, i.e., observation and interviews (individual interviews and focus groups), were used, as follows:

1) Quantitative study tools: Questionnaire

In assessing students' viewpoints regarding the integration of visual mathematics into lessons, a meticulously crafted questionnaire was developed. Administered upon the conclusion of lessons, this survey delved into the role of mathematics in cultivating both mathematical knowledge and fostering positive attitudes toward learning the subject. Consisting of 11 positively-framed paragraphs, each paired with a five-point Likert scale spanning from "strongly agree" (5) to "strongly disagree" (1), the questionnaire aimed to elicit nuanced responses, providing insights into the intricate impact of visual mathematics on students' learning experiences.

Beyond the positively framed content, the questionnaire employed a comprehensive five-point Likert scale, presenting respondents with choices ranging from "strongly agree" to "strongly disagree." The scoring system mirrored the affirmative nature of the items, assigning scores of "strongly agree" (5), "agree" (4), "neutral" (3), "disagree" (2), and "strongly disagree" (1). This strategic scoring approach was implemented to precisely capture and quantify respondents' favorable sentiments toward the presented items, ensuring a nuanced understanding of their perspectives. From the overall study sample, a substantial cohort of (121) female students emerged as the primary respondents, providing valuable insights into their perspectives on visual mathematics integration in the learning process.

The tool was verified as follows:

The validity of the questionnaire: To verify its apparent validity, the questionnaire was presented for review by 7 specialists in the field of curricula and methods of teaching mathematics, to get their observations and suggestions regarding the questionnaire and its suitability for achieving the objectives of the study, its comprehensiveness and accuracy, and assess the level of its linguistic formulation, and get any observations they deem appropriate regarding the amendment , change, or deletion. They provided valuable comments and additions to the "questionnaire" tool, which were benefited from and addressed.

The reliability of the questionnaire: To calculate its reliability, the study used Cronbach's Alpha equation, and the results are shown in Table 2.

Table 2. The reliability of the questionnaire using the Cronbach alpha coefficient

No.	Cronbach's Alpha
11	0.976

Table 2 shows that the quantitative study tool has a high level of reliability, as its reliability according to Cronbach's Alpha reached 0.976, which is a high reliability rate.

2) Qualitative study tools

a) Observation

To collect qualitative data, answer the study questions and achieve its objectives, free classroom observation was used to observe the role that visual mathematics plays in female students' interaction and performance in mathematics lesson during visual presentations of mathematics. Free classroom observation was used so that the observer does not use predetermined classifications and patterns but records his observations naturally in a fluid form and describes the reality as it happens.

b) The interview

To collect qualitative data, answer the study questions, and achieve its objectives, the qualitative interview tool was used in two places:

1) **Individual interviews**: They aimed to identify female students' opinions about visual mathematics during and after its application in mathematics lessons.
2) **Group interviews (focus groups)**: By discussing with female students their opinions of employing visual mathematics in teaching the topics of the geometry unit. Students were divided into several groups, each group includes (5–7) female students.

3) Validity and reliability

The criteria for evaluating the quality of qualitative research vary according to the nature of the research, and they replace validity and reliability in quantitative research, as pointed out by Al-Abd Al-Karim [27]. To achieve this, the following was carried out:

a) Validity

The term credibility corresponds to internal validity in quantitative research. In order to reach the validity of the data, the research team followed a number of procedures. First, they stayed at the study site during the application period of two weeks, which helped them to deepen their understanding of the phenomenon under study and to observe the participants and their behaviors and interactions during the application. In addition, they observed the phenomenon in the classroom in natural situations that reflect reality and the interaction of students with visual mathematics. They also made sure to repeat the observation process in several different sessions and chapters. Moreover, the research team carefully recorded what was seen and observed during the data collection. In addition, they used more than one data collection tool to enhance credibility; they used the interview, observation, and questionnaire tools. The research team used the language of the participants in their dialect, and included citations from that data, as it is in the presentation of the results.

b) Reliability

The term reliability corresponds to reliability in quantitative research which enables the researcher to achieve reliability in qualitative research so that subsequent researchers may repeat the research procedures. The study used a number of tools, including accurate description of study design and tools. This description included a description of the study's environment and social context and a clarification of the time and place of implementing the tools (observation and interview). This helped the researchers provide similar contexts for obtaining similar results, highlight the way to reach the results of the study, and show how the results agreed with and differed from similar previous studies.

4) Context of the study

The data of the study was collected in a public secondary school for girls in the city of Riyadh, and approval was taken from the school administration to conduct the study and interview the participants. In addition, the appropriate environment to meet the participants in the study in the school was selected, which was the classroom, the learning resource room, or one of the administrative offices in the school after having taken permission to occupy the place for some time.

5) Ethical considerations of the study

The study considered a set of ethical considerations and standards. First, the researchers obtained approval and notified the school administration of conducting the study. They informed the administration of the objectives and tools of the study, its implementation mechanism, and the expected time for implementing the study. Then, a meeting was held with the study sample before applying the study tools and materials, explaining the subject of the study, clarifying its objectives, and the method of applying the study tools. They gave the study sample freedom whether to participate or not in the study. They took permission from the sample members wishing to participate in this study and opened the way for them if they wished to withdraw at any time. The research team agreed with the study participants on the appropriate date and place for applying the study tools (observation and interviews). They sought to reassure the participants of the confidentiality of the information they provide. In order to achieve and ensure the confidentiality of the information provided by the participants, the names of the participants were encoded, and symbols were used instead of the names of the participants. The research team was also keen to take the permission of the participants when recording the interview, out of respect for their opinions and privacy. The research team also assured the participants that the data collected for the purpose of the study could not affect students' success or failure.

6) Data analysis

Although each of the methods and procedures used in the analysis of qualitative and quantitative data has different methods of analysis, the research seeks to find an integrated relationship between the data and all its tools by merging and linking them to reach the main conclusions and answer the research question.

a) Qualitative data analysis

Qualitative data analysis procedures

In this study, qualitative data collected from participants was analyzed following the steps outlined by Braun and Clarke [28]. The process is detailed below:

1) **Familiarize oneself with data:** At this initial stage, the data collected on the integration of mathematics when teaching engineering topics was thoroughly reviewed, reflected upon, and summarized. This preparation set the groundwork for determining the initial codes for participant responses.
2) **Assign preliminary codes (Descriptive Coding):** Preliminary codes were generated by the researchers during this phase. Descriptive coding, involving the summarization of passages or sentences in simpler terms, was employed. The researchers compared paragraphs and codes to ensure consistency.
3) **Search for patterns or themes:** The next stage involved sorting initial symbols, identifying similarities, and comparing them to uncover overarching themes that encapsulated all symbols.
4) **Review themes:** Themes were subjected to multiple reviews by members of the research team sequentially. A joint discussion was conducted to verify the representation of themes, ensuring they effectively captured the primary and similar symbols. Additionally, their alignment with the goals of integrating visual mathematics and their relationship to essential variables in mathematics education was assessed.
5) **Define and name themes:** In this step, the researchers identified the themes and the data they encompassed. The consistency of the themes with the data was checked to ensure accuracy.
6) **Produce report:** The final step involved the issuance of a comprehensive report. This report elucidated various topics, explaining each theme's concept and its components. It also incorporated evidence and quotations from participants' statements, along with simple statistics to substantiate the findings. The report underwent multiple reviews to ensure accuracy and clarity.

b) Quantitative data analysis procedures

The research team employed several statistical methods of the statistical package in social sciences (SPSS) as follows:

1) Cronbach's alpha reliability coefficient to ensure the reliability of the questionnaire.
2) The means and standard deviations of female students' responses to the questionnaire.

III. Results

To answer the study question, the research team collected quantitative data via a questionnaire and qualitative data was collected via observations and interviewing the study sample.

A. Results of the Quantitative Part

The research team used the questionnaire to find out female students' opinions of employing visual mathematics in mathematics lessons. The means and standard deviations of the sample's responses were calculated on each statement of the questionnaire. Table 3 shows the results.

Table 3. Female students' opinions of employing visual mathematics in teaching geometric topics

No.	Statement	mean	Standard deviation	Response
1	Visual presentations supported my understanding of the meanings of mathematical concepts and procedures.	4.66	0.534	Strongly Agree
2	Visual presentations helped to understand the mathematical concept.	4.63	0.542	Strongly Agree
3	Visual mathematics contributed to understanding and deducing various mathematical proofs.	4.66	0.516	Strongly Agree
4	Mathematical visual presentations helped me deduce properties of quadrilaterals.	4.64	0.539	Strongly Agree
5	Visual representations of mathematics enhanced my reasoning ability.	4.60	0.615	Strongly Agree
6	Mathematical visual presentations speeded up my mathematical tasks.	4.60	0.615	Strongly Agree
7	Visual presentations of mathematical concepts and procedures contributed to learning retention.	4.60	0.614	Strongly Agree
8	Visual presentations of mathematics made the learning process easy and interesting.	4.70	0.543	Strongly Agree
9	The illustrations and visuals in the textbook help me understand the lesson.	4.70	0.600	Strongly Agree
10	Visual presentations reinforced connection to real life.	4.54	0.809	Strongly Agree
11	Visual presentations contributed to my confidence in learning mathematics.	4.70	0.513	Strongly Agree
	Overall mean	4.64	0.634	Strongly Agree

Table 3 shows that the overall mean of female students' opinions of employing visual mathematics in teaching school mathematics topics was 4.64, at a level of "Strongly Agree"; the statements "Visual presentations of mathematics made the learning process easy and interesting," "The illustrations and visuals in the textbook help me understand the lesson," and "Visual presentations contributed to my confidence in learning mathematics" had the highest mean score of 4.70.

B. Results of the qualitative part

To answer the first question about female students' opinions of employing visual mathematics in geometry lesson at the secondary school, and after analyzing the results of interviews (individual interviews and focus group) and direct observation, the following topics are formed:

1) Conceptual comprehension

There is agreement about the ability of female students to provide standardized descriptions, explain quadrilateral shapes and their properties appropriately, and find the mathematical meaning and express it clearly. Participant (S1) in the 1S grade indicated that "the lesson is appropriate and wonderful and is characterized by simplicity. It deepens the meaning." The participant (S3) in class 1S said that "I realized the properties through the visual presentation. The clearest property was that the diagonal of a parallelogram divides it into two congruent triangles." Participant (S6) in class 1S reported that "I was able to write a free proof through the axioms and theorems I had learned previously and linked them to the new concepts of quadrilaterals." On the other hand, a female student stated that she wanted to remember part of these theories and properties and only expressed them orally, while another student stated that she was unable to remember the properties and theories; the videos were repeated again, and she expressed her understanding in writing in her own simple way and excelled in that.

2) Visual thinking

There is a large degree of agreement about female students' ability to deal with and visually recognize quadrilateral geometric shapes and, as well as explain the ambiguity in finding the sum of the measures of the interior angles of a polygon and express them and their ideas visually and verbally. For example, participant (S2) in grade 1S indicated that "the interactive movement of the angles of sides, diagonals, and exterior and interior angles made me solve higher-order thinking skills easily." Participant S4 in grade 1S stated that "the presentation is interesting and attractive, especially with the presence of colors that attract the viewer and increase my geometric knowledge." Participant (S5) in grade 1S added that "watching the video with its interactive movements is sufficient for the usual illustrative explanation." On the other hand, two female students stated that the presentation was somewhat suitable, and they would like to add interactive games that support the properties of quadrilaterals in the visual presentations.

3) Mathematical appreciation

There is a significant agreement among female students about their understanding and interpretation of mathematical situations in geometry, especially in quadrilaterals and their properties, as abstract mathematical concepts are linked to reality and sensory actions. Participant (S1) in grade 1S indicated that "I remember the law of finding the sum of the measures of the interior angles of any polygon easily, and I cannot not forget it." Participant (S3) in grade 1S stated that "The visual presentation has provided me with a great capability of deep understanding, inference and their application in ddoing technical creative performance tasks using the Canva program." Participant (S5) in grade 1S adds that "the visual presentation is appropriate and full of information, which made me link it to examples from my local environment and understand them well." Participant S6 in grade 1S also added that "the presentation is creative and short and a useful reference for study and revision".

On the other hand, a female student stated her desire to explain the lesson again to her classmates in her own way, using the interactive whiteboard in the resource room. Her persistent wish was fulfilled. By the end of her lesson, she asked her colleagues to make an interactive conceptual map and send it via Microsoft Teams (Fig. 2).

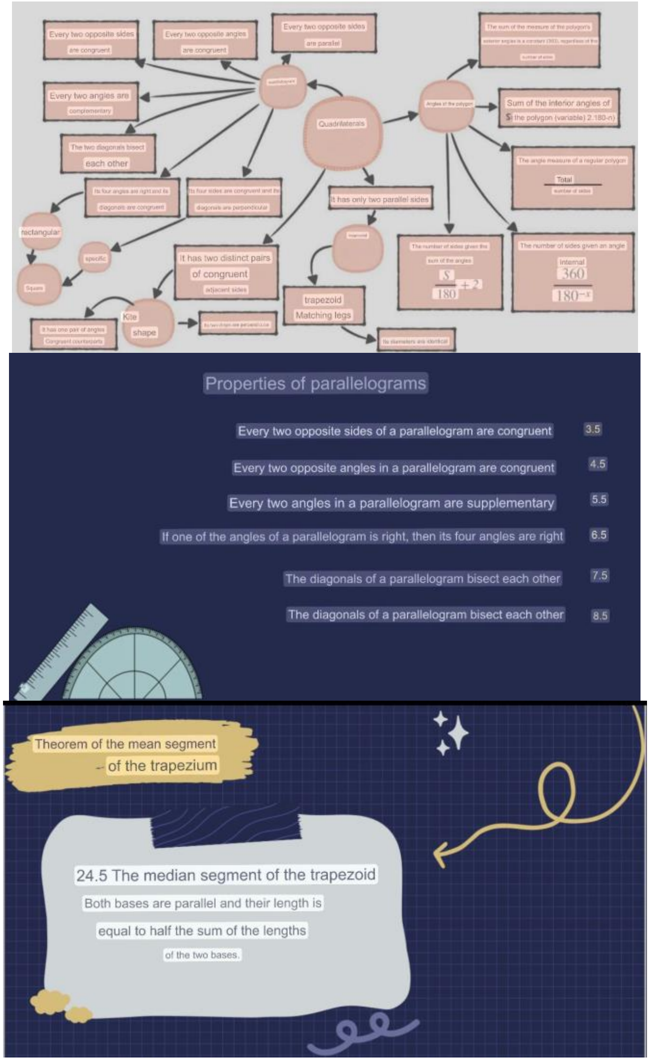

Fig. 2. Examples of extra-curricular activities of female students.

4) *Mathematical connection*

There is an agreement about the ability of students to connect different mathematical ideas and show how they are built on each other to produce new structures. Female students applied geometrical concepts in making brochures on quadrilaterals. They also applied them in various school subjects and life applications. Participant (S2) in class 1S said that "adding interactive colors to all the measurements of interior angles helped me to make distinguished presentations with which I can attract the attention of my classmates and explain topics clearly in the Computer school subject." Participant (S4) in grade 1S stated that "the visual video helped me understand the important and internal details of quadrilaterals. Using them, I can innovate and make simple mathematical devices such as jumping and other sports." Participant (S6) in grade 1S added that "the educational video made me want to do volunteer work during the Hajj season to mark the columns and the heads of the parallelograms of the pilgrims' tents in Mina." Another student reported that "I realized the value of diamonds and precious stones through my deep understanding of the properties of rhombuses and squares, and through my ability to distinguish, express opinions and observations, and calculate them mathematically." On the other hand, a student reported that, after the visual presentation of quadrilaterals, she could present math boards on the school activity day in the form of scientific posters, because she realized all the characteristics that help in that.

C. Comparison and Linking

The stage of comparing the quantitative and qualitative results in the same study to provide an integrated and coherent picture of the study is one of the complex and important stages. Reviewing the quantitative and qualitative results reveals their consistency, as visual presentations enhanced female students' comprehension of mathematical concepts, understanding the relationship between concepts and linking them. Visual presentations motivated female students to infer mathematical knowledge in geometry topics and to think and be creative in presenting various designs, as there is variety in presenting designs such as summaries of lessons. It also contributed to enhancing the ability of female students to link mathematics and other subjects, and to relate it to the daily life of the learner. The emotional aspect is one of the important aspects in teaching and learning mathematics. Visual presentations of geometry topics contributed to improving the students' attitudes towards learning mathematics and raising the level of self-confidence in their learning, as well as contributing to the enjoyment of learning various geometry topics.

IV. Discussion

The study reached many important results regarding teaching geometry topics through visual representations of mathematical knowledge. Visual presentations contributed to the development of the understanding of mathematical concepts of geometry topics, and this may be due to the attractiveness of the presentations and their focus when presenting the lessons. They presented mathematical content through multiple sensory audio-visual channels with a sense of movement. They helped learners to visualize mathematical knowledge and understand mathematical drawings and shapes. Rotger & Ribera [29] indicated that the visual perception of the mathematical problem supports the student's ability to solve mathematical issues and problems. On the other hand, visual (video) presentations, due to the advantages of technology, provided female students with the opportunity to repeatedly revise mathematical knowledge. The result of the study agrees with that of Goda [30] which demonstrated the effectiveness of using the Geogebra program in teaching geometry and spatial reasoning in developing conceptual comprehension, and with that of Al-Omari [31], which revealed the effectiveness of designing and teaching educational units according to the STEM approach in developing conceptual comprehension.

The study also revealed the effect of visual presentations on the development of thinking skills in general and visual thinking skills in particular. This may be attributed to the role of visual presentations of mathematical knowledge in activating mental capabilities via imagination and mental images of things and experiences. Visual thinking is a pattern of thinking that is closely related to geometry, and it trains the learner to discover some of the relative relationships that may appear from the analysis of geometric shapes. This is confirmed by Aamer and Al-Masry [1] who say that Van Hiele's theory of geometric thinking contained a set of skills based on thinking through form or the visual model. This result is consistent with the study of Metwally *et al*. [32] regarding the effect of using electronic mental maps to teach geometry on developing visual thinking skills.

The study concluded that visual presentations contributed to the development of mathematical connection skills. This is due to sequencing the presentation of mathematical knowledge in each lesson and the inclusion of situations from the surrounding environment. The result of current study is consistent with that of Al-Khalili and Al-Baalouji [33] which revealed the effect of Lesh model of multi-representations on developing mathematical connection, and that of the study of Ali *et al.* [34], which showed the effect of using the school wide optimum mode (SWOM) strategy in teaching geometry on the development of mathematical connection skills.

The study also led to the development of female students' mathematical appreciation. This may be due to the fact that the diversity and multiplicity of ideas in the presentations in a focused manner contributed to presenting mathematical knowledge in a contextual manner related to its applications in students' lives. Moreover, visual presentations provided educational experiences in an interesting visual form that highlight mathematical beauty in a way that achieves fun for students while learning mathematics and generates an internal motivation for them to learn it, taking advantage of the innate need for beauty in humans. This is confirmed by Goldin-Meadow [35] and Abbas [36] which indicated that providing educational experiences that have fun and an aesthetic sense helps to integrate students with mathematical experiences and make them realize and appreciate the beauty of mathematics.

V. Conclusions

Visual mathematics is a recent trend in the field of teaching and learning mathematics. It relies on presenting mathematical knowledge in the form of multiple representations and provides an opportunity to recognize the mathematical relationships between the elements of mathematical knowledge. The focus on geometry topics is appropriate and consistent with the trend. On the other hand, the use of visual mathematics in the secondary school is appropriate for the age group and the nature of the learners, as students at this stage become able to take responsibility and to discuss and design different models and representations to summarize studied mathematical knowledge.

The study also confirms the importance of teachers designing mathematical representations that take into account the school stage and the appropriate visual design features in terms of technical or scientific aspects.

Due to the study's utilization of a mixed-method and parallel design, it encountered several limitations. The research was constrained to a single study unit, spanning only three weeks. Additionally, a challenge emerged in managing an increased number of female students within one class during the implementation of the study, particularly in the context of presenting and discussing the videos. To ensure inclusivity, the researchers conducted interviews during class time, providing an opportunity for everyone to participate. However, limitations were present as focus group sessions were restricted to a single session. Another challenge arose in accommodating individual discussions, with only six female students participating, representing two students from each class. The research team proposes a group of studies dealing with visual mathematics. For example, they propose conducting a study of students' opinions about the employment of mathematics in other areas of school mathematics such as algebra, statistics, probability, etc., and in other school stages. Another proposal is to investigate the effect of employing visual mathematics on a number of important variables in teaching mathematics, such as mathematical prowess, geometrical thinking, etc. A third proposal is to study teachers' perceptions of employing visual mathematics and the training needs of teachers to activate visual mathematics. A fourth proposal is to build training programs to train teachers to design visual presentations to teach mathematical knowledge. A fifth proposal is to conduct a study to present a proposal to employ visual mathematics in school mathematics curricula. Finally, it is proposed that a study should be conducted on the readiness of educational environments to employ visual mathematics.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Conceptualization, I.K. and Y.W. and M.A.; methodology, T.W.; software, I.K. and M.A.; validation, T.W. and Y.W.; formal analysis, I.K.; investigation, Y.W.; resources, T.A.; data curtain, I.K.; writing—original draft preparation, M.A. and I.K.; writing—review and editing, Y.W. and I.K.; visualization, I.K.; supervision, M.A.; project administration, T.W.; funding acquisition, I.K. All authors have read and agreed to the published version of the manuscript."

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