Game-Based Adaptive Learning in Probability Education

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***Abstract*—Traditional approaches often fall short in effectively teaching complex subjects like probability and dynamic programming, especially in contexts requiring high engagement and individualized learning paths. This paper presents a simulation-based experimental study exploring the potential of an Artificial Intelligence (AI) adaptive learning system through the development of a game-based learning tool. The system utilizes dynamic programming principles and decision tree regressors to adjust the game complexity in real-time, based on simulated student performance. The adaptive dice game provides personalized learning experiences that improve both engagement and comprehension of key mathematical concepts. The experiment evaluates how adaptive difficulty settings influence strategic decision-making and learning outcomes. The results demonstrate that adaptive learning systems can significantly enhance mathematical education by offering customized learning paths that improve understanding of complex concepts. This study contributes to the discussion on the potential of integrating AI with educational technologies to enhance learning outcomes, particularly in disciplines that demand high analytical skills.**

***Keywords*—game-based learning, adaptive learning, Artificial Intelligence (AI), probability**

I. Introduction

Probability is a cornerstone of mathematical and statistics education, underpinning diverse disciplines from actuarial science to computer algorithms. Mastery of probability not only enhances analytical skills but also equips students to tackle complex decision-making challenges across various scientific and industry sectors. Despite its importance, probability remains a challenging subject for many students due to its abstract concepts and the need for strong problem-solving skills.

Traditional approaches to teaching probability often rely heavily on theoretical instruction and static problem sets that fail to engage students or cater to individual learning needs. Such methods can be particularly limiting, as they do not adjust to the varying paces and styles of student learning, potentially leading to disengagement or a lack of deep understanding. Modern educational theories such as Social Learning Theory [1] suggest that students learn more effectively when actively engaged and when learning is contextualized through meaningful experiences.

Game-based learning is aligned with Constructivist approaches by providing students with an interactive and experiential way to understand concepts like probability. Vygotsky's Zone of Proximal Development (ZPD) also highlights the importance of providing learners with challenges that are within reach but still slightly beyond their current capabilities, requiring guidance or adaptive support [2]. Adaptive learning systems, supported by Cognitive Load Theory [3], reduce unnecessary cognitive strain by adjusting the difficulty of learning tasks in real time, thus optimizing the learner's engagement and comprehension.

To address these educational challenges, interest in leveraging technology has been growing to create more dynamic and engaging learning environments. Game-based learning, which combines educational content with interactive game elements, has emerged as a particularly promising approach [4, 5]. However, while game-based learning increases engagement, not all systems adapt to the individual learner's progress and needs. This paper introduces an innovative solution combining game-based learning with personalization to teach probability powered by Artificial Intelligence (AI). By embedding intelligent adaptive mechanisms within a dice game, the system dynamically adjusts the difficulty and nature of probability problems based on real-time analysis of student performance. This personalized approach not only aims to enhance engagement but also to improve understanding of probability and dynamic programming through tailored educational experiences.

The primary objective of this study is to evaluate the effectiveness of the AI-driven adaptive dice game in enhancing students' understanding of probability. By providing a detailed case study, this research aims to contribute to the broader discourse on adaptive learning technologies in education, particularly in the teaching of complex mathematical and computational concepts. Previous studies [6] have explored the application of AI-enabled adaptive learning systems across various subjects, demonstrating their effectiveness in diverse educational contexts, particularly in math [7, 8]. This research builds on that foundation by focusing specifically on the use of adaptive learning in probability and dynamic programming education. The findings are intended to inform educators and technologists about the potential of integrating AI with game-based learning to create highly effective educational tools.

II. Literature Review

Technology-enhanced adaptive learning and personalized learning remain a prominent area of interest. This is largely driven by the utilization of personalized data such as student preferences, achievements, profiles, and learning logs, with promising potential for AI integration [9]. In mathematics, adaptive teaching has benefited from student stimuli, teacher reflection and metacognition, and teacher action [10]. There is evidence that the adaptive learning approach enhances student learning in cognitive performance analysis [11, 12].

Game-Based Learning aligns with Constructivist approaches to education, emphasizing the active role of learners in constructing their own knowledge through

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interactive and experiential methods. According to Social Learning Theory, learning is most effective when students are actively engaged and learning is contextualized through meaningful experiences [1]. Vygotsky's ZPD [2] highlights the need for learning experiences that are just within reach of the learner's current abilities but still require guidance and support, a core principle behind adaptive learning systems. Game-based learning, which integrates educational content with interactive game elements, has shown promise in increasing student engagement [4, 5]. However, traditional game-based learning systems do not always adapt to individual student progress. Adaptive learning, when integrated with game-based systems, addresses this gap by tailoring the learning experience in real-time to each student's needs, enhancing the effectiveness of the game-based approach [13]. In [14], an adaptive and intelligent e-learning system was developed to individualize learning in secondary school probability subjects. Their system utilizes the VAK (Visual–Auditory–Kinesthetic) learning styles model to tailor content delivery to individual student preferences, offering dynamic adjustments based on learners' performance. This adaptive approach aligns with the broader goal of enhancing learning outcomes through personalized learning environments, a core focus of this study. Moreover, the system incorporates Maximum Likelihood Estimation to assess student performance and adjust the difficulty level of tasks accordingly. By adapting to the individual learning style and progress of each student, it serves as an example of how personalized learning paths can be effectively incorporated into game-based and adaptive systems.

The combination of game-based learning, adaptive learning, and real-time feedback forms the core of this study's approach, as illustrated in Fig. 1. This theoretical framework underscores the importance of providing students with dynamic, engaging, and personalized learning experiences, ultimately aiming to improve their motivation and active participation in learning activities.

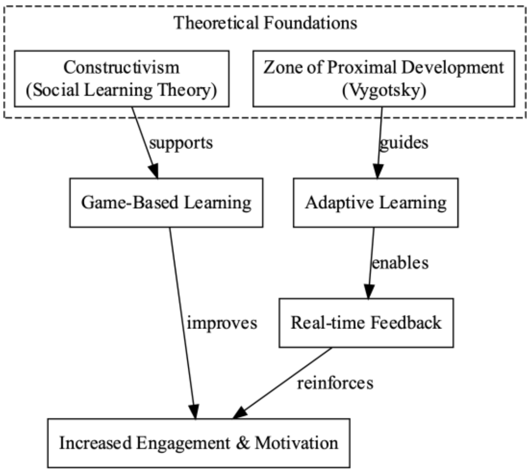

Fig. 1. Theoretical framework of game-based adaptive learning.

Recent research highlights the need for more adaptive gamification frameworks in online training environments, where static player profiles are commonly used to tailor experiences [15]. In addition, serious games have shown significant promise in formal education, particularly in enhancing student engagement and learning outcomes [16]. This underscores the broader applicability of game-based learning in different STEM fields, showing that serious games can drive engagement and cognitive outcomes in various educational contexts.

With AI increasingly being adapted, it can enhance the adaptive learning system [17] and provide personalized support in mathematics education [18]. When combined with game-based learning, adaptive computer games have shown better outcomes than conventional approaches, especially for students with higher prior learning attitudes [19]. Personalized educational computer games not only promote learning motivation but also improve the learning achievements of students [20]. AI can potentially be incorporated in assessments to provide a more flexible and student-centered approach [21]. This research aims to contribute to the broader discourse on adaptive learning technologies in education, particularly in teaching complex mathematical and computational concepts, by providing a detailed case study. The findings are intended to inform educators and technologists about the potential of integrating AI with game-based learning to create highly effective educational tools.

III. Methods

This study employs a simulation-based experimental design to evaluate the effectiveness of an AI-driven adaptive learning system integrated with a dice game. The research focuses on a proof-of-concept to demonstrate the potential of adaptive learning systems, grounded in principles of Item Response Theory (IRT) [22], in teaching probability and dynamic programming concepts to university-level students studying mathematics, computer science, or related fields at the undergraduate or early postgraduate level.

The dice game developed for this study is designed to teach key concepts in probability and dynamic programming through an engaging and interactive experience. As shown in Fig. 2, the flow of the game begins by initializing the game and rolling the dice. Players then decide which dice to freeze or reroll. The system uses dynamic programming to compute the optimal decisions at each step, providing real-time feedback to players based on their choices. Following the principles of IRT, the game's difficulty adapts dynamically based on player performance, similar to how IRT adjusts the complexity of questions based on learner ability. In this system, a decision tree model adjusts the game's difficulty by changing the number of dice or modifying the complexity of decisions for subsequent rounds. This mirrors the IRT approach where the difficulty of the next task or question is determined by the learner's prior responses [14].

This adaptive process continues through several rounds, where the game's difficulty is dynamically tailored to the player's performance. By using the principles of IRT to adjust the game in real-time, the system ensures that each player experiences a personalized learning journey, with challenges aligned to their current ability level. As described in [14], this approach provides a more individualized and effective learning environment, promoting student engagement and mastery of key concepts in probability and dynamic programming.

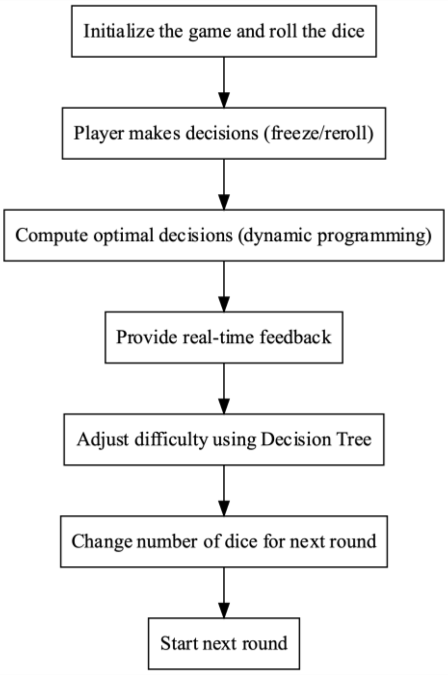

Fig. 2. Interactive dice game workflow.

A. *Game Mechanics: Dynamic Programming*

The dice game serves as an interactive tool for students to grasp probability and dynamic programming concepts through hands-on gameplay. Inspired by the example provided in [23], the game mechanics focus on maximizing the face value of frozen dice using dynamic programming principles. By making strategic decisions about freezing or rerolling dice, players are exposed to probabilistic thinking and optimization strategies. The game is designed to progressively build the student's understanding of how dynamic programming applies to decision-making under uncertainty, promoting deeper engagement with mathematical and computational concepts.

We define the i-th state of the maximization dice game as when the player is throwing i ordinary fair dice simultaneously, where $1 \leq i \leq 5$. Let $(F_1, F_2, ..., F_i)$ be a possible outcome of face values of the dice in this state. Note that there are 6^i distinct possible outcomes.

Given any possible outcome $(F_1, F_2,...,F_i)$ in the i-th state, we first sort the elements in ascending order and denote the ordered outcome as $(F_{(1)}, F_{(2)} ,...,F_{(i)})$.

The rule of the game specifies that the player has to freeze at least one die, and one frozen die must be $F_{(i)}$, the largest one. Then, the player needs to make a decision on the additional (if any) number of dice to be frozen, and the rest of them will be rerolled. Denote k as the number of dice to be rerolled ($0 \leq k \leq i-1$). After deciding the value of k, the player will reroll the smallest k dice $(F_{(1)}, F_{(2)} ,...,F_{(k)})$ and move to the state with k dice left.

We let $S_{i,k}$ be the random variable of the total face value on the dice if the player throws i dice and chooses to reroll k dice. Let S^* be the random variable of the total face value on the dice if the player throws i dice, chooses to reroll k dice, and then follows an optimal strategy with the remaining k dice. Let S^* be the random variable of the total face value on the dice if the player throws i dice and follows an optimal strategy for freezing dice.

The game first starts with $i = 5$, and the last possible state is $i = 1$. Following the principle of dynamic programming, we start determining the optimal strategy and expected value from the last possible state and work backward to the earlier states.

1. With 1 Die:
   $E(S^*_1) = (1+2+3+4+5+6)/6 = 3.5$.
2. With 2 Dice:
   $E(S^*_{2,0} \mid F_1,F_2) = F_{(1)}+F_{(2)}$ if no reroll;
   $E(S^*_{2,1} \mid F_1,F_2) = 3.5+F_{(2)}$ if reroll

The player chooses to reroll if $F_{(1)} < 3.5$. The expected value is average of all possible outcomes.

3. With 3 Dice:
   $E(S^*_{3,0} \mid F_1,F_2,F_3) = F_{(1)}+F_{(2)}+F_{(3)}$ if no reroll;
   $E(S^*_{3,1} \mid F_1,F_2,F_3) = 3.5+F_{(2)}+F_{(3)}$ if reroll 1 die;
   $E(S^*_{3,2} \mid F_1,F_2,F_3) = 8.2361+F_{(3)}$ if reroll 2 dice.

The expected value is averaged across all scenarios where decisions maximize the score based on the calculated expected values.

4. With 4 Dice:
   $E(S^*_{4,0} \mid F_1,F_2,F_3,F_4) = F_{(1)}+F_{(2)}+F_{(3)}+F_{(4)}$ if no reroll;
   $E(S^*_{4,1} \mid F_1,F_2,F_3,F_4) = 3.5+F_{(2)}+F_{(3)}+F_{(4)}$ if reroll 1 die;
   $E(S^*_{4,2} \mid F_1,F_2,F_3,F_4) = 8.2361+F_{(3)}+F_{(4)}$ if reroll 2 dice;
   $E(S^*_{4,3} \mid F_1,F_2,F_3,F_4) = 13.4249+F_{(4)}$ if reroll 3 dice.

The expected value is calculated similarly by averaging across all outcomes, taking the maximum expected value for each scenario.

5. With 5 Dice:
   $E(S^*_{5,0} \mid F_1,F_2,F_3,F_4,F_5) = F_{(1)}+F_{(2)}+F_{(3)}+F_{(4)}+F_{(5)}$ if no reroll;
   $E(S^*_{5,1} \mid F_1,F_2,F_3,F_4,F_5) = 3.5+F_{(2)}+F_{(3)}+F_{(4)}+F_{(5)}$ if reroll 1 die;
   $E(S^*_{5,2} \mid F_1,F_2,F_3,F_4,F_5) = 8.2361+F_{(3)}+F_{(4)}+F_{(5)}$ if reroll 2 dice;
   $E(S^*_{5,3} \mid F_1,F_2,F_3,F_4,F_5) = 13.4249+F_{(4)}+F_{(5)}$ if reroll 3 dice;
   $E(S^*_{5,4} \mid F_1,F_2,F_3,F_4,F_5) = 18.8436+F_{(5)}$ if reroll 4 dice.

The value is calculated by averaging over all potential outcomes, where the decision in each case is chosen to maximize the sum of the dice kept and the expected values of the dice rerolled.

Table 1 summarizes the expected values for each number of dice. The series of calculation involves a comprehensive assessment of all possible combinations of dice outcomes to determine the best strategy statistically. This methodology uses a large sample of simulated rolls or a full combinatorial analysis to calculate the average. By strategically considering which dice to freeze and which to reroll, the player maximizes their expected score based on the combination of immediate values and the potential improvement from rerolls. This strategic depth illustrates the application of dynamic programming and probabilistic thinking in decision-making processes.

Table 1. Expected values for each number of dice

Number of Dice	Expectation
0 Die	$E(S^*_0) = 0$
1 Die	$E(S^*_1) = 3.5$
2 Dice	$E(S^*_2) \approx 8.2361$
3 Dice	$E(S^*_3) \approx 13.4249$
4 Dice	$E(S^*_4) \approx 18.8436$
5 Dice	$E(S^*_5) \approx 24.4361$

B. Game-Based Learning, AI Adaptivity and Feedback

The dice game simulation serves as an interactive tool for teaching and reinforcing decision-making under uncertainty. In the game, players start each round by rolling a predefined number of six-sided dice. Following the roll, players face the strategic decision of which dice to keep the current face value and which to reroll in hopes of achieving a higher score. This decision-making process is underpinned by dynamic programming principles, which break down complex decisions into simpler, recursive subproblems. At each stage, the game calculates expected values for different potential actions, guiding players toward the most statistically advantageous choices.

A core feature of the game is its AI-driven adaptivity, which customizes the difficulty level based on the player's performance. Using machine learning techniques, particularly a Decision Tree Regressor, the AI evaluates each player's past decisions, scores, and outcomes. If a player consistently performs well, the game dynamically increases the complexity by modifying the number of dice or introducing more challenging decision scenarios. Conversely, if a player struggles with suboptimal decisions, the AI reduces the complexity, creating a more manageable learning experience. This ensures that the game remains both engaging and appropriately challenging, preventing frustration or boredom and encouraging continuous learning.

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Initialize:
- Set number of dice.
- Calculate expected values for each
possible dice count.

For each round of the game:
  1. Roll the dice.
  2. Compute the optimal decision using
dynamic programming:
     - For each subset of dice (frozen
candidates):
       - Calculate potential score from frozen
dice.
       - Estimate future score using expected
values of rerolling the rest.
       - Keep track of the decision leading to
the highest score.
  3. Input player's decision.
  4. Reroll non-frozen dice and calculate the
final score for the round.
  5. Provide feedback comparing the
player's decision to the optimal decision.
  6. Record the player's score.
  7. Adjust the difficulty based on the
player's performance history using the
decision tree:
     - Input the sequence of scores to the
model.
     - Predict the optimal number of dice for
the next round based on the model's output.
     - Adjust the number of dice to either
increase or decrease the game's difficulty.

Repeat for the desired number of rounds or
until a stopping condition is met (e.g., a
performance threshold).
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Fig. 3. Pseudo algorithm.

Feedback is integral to the game's learning process. At the end of each round, the game provides real-time feedback by comparing the player's decisions with the optimal choices calculated by the game's algorithms. This immediate feedback helps players understand the effectiveness of their decisions, offering corrective suggestions where necessary and reinforcing successful strategies. By directly connecting players' actions with theoretical principles, the feedback mechanism enables reinforcement learning, helping players to internalize optimal decision-making techniques quickly.

The feedback system plays a crucial role in strategic improvement. By reflecting on the feedback provided, players can adjust their strategies in future rounds, fostering a deeper understanding and application of dynamic programming. This iterative learning process is key to mastering complex decision-making skills that dynamic programming teaches, making the feedback an invaluable part of the educational experience.

The integration of game mechanics, AI-driven adaptivity and real-time feedback ensures a personalized, engaging, and educational experience, as illustrated in Fig. 3, which presents a pseudo algorithm outlining the flow of decision-making and feedback within the game. The adaptive difficulty system adjusts the learning curve to match each player's evolving skill level, while the feedback mechanism reinforces correct strategies, making the game an effective tool for teaching probability and dynamic programming.

IV. RESULT

This section focuses on the application and outcomes of a dice game designed to teach dynamic programming principles and decision-making strategies through real-time feedback and adaptive difficulty settings. The results from several rounds of gameplay are discussed below to illustrate player interactions with the game mechanics and the educational feedback provided.

In Round 1 (see Fig. 4), the player chose to freeze two dice, including a die with a lower value, which led to a suboptimal final score of 9. Feedback suggested freezing only the highest die (5) for a potentially higher score. This round provided an important learning opportunity for assessing which dice to freeze in order to maximize potential future scores.

Round 1:
- Initial Roll: [1, 5, 1]
- Player Decision: Chose to freeze two dice ([5, 1]), rerolled one die ([3]).
- Final Score: 9
- Feedback: Advised to freeze only the highest die (5) for a potentially higher score.

Fig. 4. Player decisions, final score and feedback

In Round 2 (see Fig. 5), the player corrected their approach by freezing the highest die (6) and rerolling the others. This resulted in a higher final score of 14. The feedback praised this decision as it reinforced the correct application of dynamic programming principles, showing how optimal decision-making can directly improve outcomes.

However, in Round 3 (see Fig. 6), the player reverted to freezing both dice, including a lower value die. As a result, the final score was 8. The feedback suggested freezing only the highest die (6) to improve future performance, illustrating a missed opportunity to further apply the optimal strategy.

Round 2:
- Initial Roll: [3, 6, 3]
- Player Decision: Correctly chose to freeze the highest die (6) and reroll the others ([4, 4]).
- Final Score: 14
- Feedback: Praised for making the optimal choice.

Fig. 5. Optimal decision and reinforcement of strategy

Round 3:
- Initial Roll: [2, 6]
- Player Decision: Froze both dice.
- Final Score: 8
- Feedback: Suggested freezing only the highest die (6) to enhance scoring potential.

Fig. 6. Suboptimal decision and suggested improvements

By Round 4 (see Fig. 7), a pattern of freezing more dice than optimal emerged, as the player froze both dice again, resulting in a score of 7. The feedback recommended freezing only the highest die (4), which highlighted the ongoing learning curve in the decision-making process.

Round 4:
- Initial Roll: [4, 3]
- Player Decision: Froze both dice.
- Final Score: 7
- Feedback: Suggested freezing only the highest die (4) for better outcomes.

Fig. 7. Learning curve indicated by suboptimal freezing

In Round 5 (see Fig. 8), the player demonstrated improvement by making the optimal decision to freeze both high-value dice and reroll the lowest die. This decision resulted in a final score of 11, serving as positive reinforcement of the dynamic programming strategies previously introduced.

Round 5:
- Initial Roll: [4, 1, 4]
- Player Decision: Made the optimal choice by freezing the 4s and rerolling the 1 ([3]).
- Final Score: 11
- Feedback: Praised for making the optimal choice.

Fig. 8. Learning curve indicated by suboptimal freezing

Round 6 (see Fig. 9) presented a dilemma for the player, with multiple high-value dice rolled. The player decided to freeze three of the four high dice, rerolling one. While this led to a high final score of 23, feedback indicated that freezing all dice would have been a more optimal decision, further emphasizing the balance between risk-taking and optimal strategy.

In Round 7 (see Fig. 10), the player showed notable strategic improvement. The decision to freeze the highest die (4) and reroll the others resulted in a final score of 12. This round demonstrated the player's adaptation to previous feedback and a more refined application of decision-making principles.

Round 6:
- Initial Roll: [6, 6, 4, 6]
- Player Decision: Froze three dice, rerolled one ([5]).
- Final Score: 23
- Feedback: Advised to freeze all dice next time as they were all high.

Fig. 9. Learning curve indicated by suboptimal freezing.

Round 7:
- Initial Roll: [1, 1, 4]
- Player Decision: Efficiently froze the highest (4) and rerolled the others ([6, 2]).
- Final Score: 12
- Feedback: Praised for making the optimal choice.

Fig. 10. Strategic improvement and adaptation

The simulation rounds provide substantial evidence that the game effectively promotes understanding and application of dynamic programming principles through interactive gameplay and adaptive challenges. The feedback mechanism proved essential in guiding players towards better strategies, facilitating a deeper understanding of probabilistic outcomes and strategic decision-making. Over time, players demonstrated improved decision-making abilities, aligning more closely with the optimal strategies suggested by the game's AI-driven analysis.

V. Discussion

This study employed a simulation-based experimental design to explore the effectiveness of an AI-driven adaptive learning system integrated with a dice game, aimed at teaching probability and dynamic programming concepts. The simulated gameplay provided insights into how players make decisions, adapt their strategies, and respond to real-time feedback in an adaptive learning environment. The subject of this study involved virtual players interacting with the game under controlled conditions, making it a proof-of-concept for adaptive learning systems in education.

Throughout the simulation, players exhibited a range of decision-making patterns, consistent with findings in other adaptive learning research. Learners initially tend to adopt conservative strategies in Rounds 1, 3, and 4 where players froze more dice than necessary, prioritizing guaranteed outcomes over potential future gains. This behavior reflects the risk-averse tendencies often seen in uncertain decision-making scenarios, where individuals tend to avoid risk when potential losses are uncertain [24].

A key feature of the game is its ability to adjust probabilities based on player performance, adding a layer of complexity and personalization. By increasing the difficulty for high-performing players and lowering it for those struggling, the game fosters a personalized learning environment. This approach closely aligns with Vygotsky's ZPD, which suggests that learners benefit most when they are presented with tasks that are within their reach but still slightly beyond their current level of competence, requiring guidance or support [2].

The AI-driven game acts as a form of scaffolding, providing players with real-time feedback and gradually increasing challenges that match their evolving skill levels. As a result, students are supported through their ZPD, enhancing their ability to grasp complex concepts such as probability and dynamic programming more effectively. This method is consistent with research highlighting the importance of adaptive feedback in maintaining engagement and promoting learning.

Incorporating IRT-based adaptivity into the game offers more than just real-time feedback; it provides a tailored assessment of each learner's ability. Unlike traditional game-based learning systems, which present identical tasks to all learners, this system dynamically adapts by selecting problems based on prior performance. Players with higher ability are presented with more complex scenarios to challenge their understanding, while those struggling receive simpler tasks to build confidence. This approach aligns with [14], where adaptive learning systems are shown to improve educational outcomes by continuously personalizing the learning pathway based on each student's progress.

The results from this study suggest that AI-driven adaptive learning systems like the dice game hold significant promise in educational settings, particularly for teaching complex mathematical concepts such as probability, statistics, and optimization. The engaging nature of the game aligns with the findings of [5], which demonstrate that game-based learning can make difficult subjects more accessible to a broader range of learners by catering to different learning styles. Furthermore, the adaptivity introduced by the AI component ensures that students at various skill levels benefit from the tool, consistent with the findings of [17], which discuss the role of AI in creating more inclusive educational experiences.

VI. LIMITATION

While the results are promising, several limitations must be acknowledged. First, the sample size of game rounds used in this research is relatively small. A larger sample size and a more diverse set of gameplay data would provide a more robust dataset, thereby enhancing the reliability and generalizability of the conclusions. Additionally, expanding the study to include comparative analyses, such as comparing the outcomes of this AI-driven adaptive learning system with other game-based learning tools or traditional teaching methods, could further validate the effectiveness of the approach.

The study is primarily a theoretical exploration, focusing on conceptualizing an AI-driven adaptive learning tool through a proof-of-concept simulation. As a result, no real-world data has been collected, and the findings are based on simulated gameplay rather than qualitative or quantitative data from actual participants. While the simulation provides insights into how adaptive mechanisms may influence learning behaviors and decision-making, the lack of empirical testing limits the generalizability of the results. Qualitative data, such as player engagement, motivation, and perceived difficulty, could provide deeper insights into the subjective aspects of game-based learning. Understanding these elements is crucial for designing more effective and user-friendly educational tools.

Future research should aim to address these limitations by increasing the sample size, incorporating comparative studies, and examining the effects of varying player characteristics to better understand how different learners interact with and benefit from adaptive game-based learning environments with real-world data. Additionally, incorporating both qualitative and empirical data can further validate the findings and explore how these adaptive mechanisms affect real learners in diverse educational contexts.

VII. CONCLUSION

This study explored the integration of AI with game-based learning through the development of an adaptive dice game designed to enhance the understanding of probability and dynamic programming. The game utilized an AI-driven system, specifically a Decision Tree Regressor, to dynamically adjust the complexity of problems based on real-time analysis of student performance. This approach aimed to provide personalized learning experiences that adapt to individual skill levels and learning paces, thus optimizing the educational impact.

The results of this study indicate that the adaptive dice game was effective in promoting engagement and enhancing comprehension of complex mathematical concepts. The real-time feedback mechanism played a crucial role in guiding players toward better strategies and deeper understanding, as evidenced by the improvement in decision-making abilities across successive rounds of gameplay. Players who initially exhibited risk-averse behaviors and suboptimal strategies were able to align their decisions more closely with optimal strategies as they progressed, demonstrating the potential of adaptive learning environments to facilitate meaningful educational outcomes.

The AI-driven adaptivity ensured the game remained appropriately challenging for players of varying skill levels, preventing both frustration and boredom. This dynamic adjustment maintained high levels of engagement and promoted continuous learning, showcasing the benefits of personalized learning paths in educational technologies.

In conclusion, this research underscores the potential of intelligent adaptive learning systems to revolutionize educational methodologies. By integrating AI with game-based learning, educators can create highly effective and engaging tools that not only enhance understanding of complex concepts but also cater to individual learning needs. The implications of this study extend beyond probability education, suggesting broader applications in various mathematical and computational fields. This work paves the way for future research and development in adaptive educational technologies, aiming to create more inclusive, engaging, and effective learning environments.

CONFLICT OF INTEREST

The author declares no conflict of interest.

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