Integrating GeoGebra into a Propose-STEM Learning Model to Enhance Mathematical Critical Thinking in Engineering Mathematics

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Manuscript received April 2, 2026; revised May 18, 2026; accepted June 29, 2026; published November 12, 2026

***Abstract*—Developing mathematical critical thinking is an essential competency for engineering students. Based on observation, mathematics instruction in many engineering programs still emphasizes procedural problem-solving rather than higher-order thinking. Consequently, students often struggle to interpret problems, evaluate solution strategies, and justify mathematical reasoning. This study examined the effectiveness of integrating GeoGebra into a Propose-STEM learning model to enhance students' mathematical critical thinking in engineering mathematics courses. The Propose-STEM model combines problem-based learning and problem posing within a STEM-oriented environment, supported by dynamic visualization and interactive exploration using GeoGebra. A quasi-experimental design was conducted with undergraduate engineering students enrolled in a calculus course. The experimental group was taught using the GeoGebra-supported Propose-STEM model, while the control group was trained through conventional instruction. Data were collected through pre- and posttests of mathematical critical thinking and analyzed using normalized gain scores, independent-samples t-tests, and Analysis of Covariance (ANCOVA). The results show that the experimental group achieved significantly greater improvement than the control group, with a higher normalized gain (0.70) than the control group (0.52). After controlling for baseline differences using ANCOVA, the instructional effect remained statistically significant, with a moderate-to-large adjusted effect size (partial $\eta^2 = 0.270$). These findings indicate that integrating GeoGebra into the Propose-STEM framework may create a meaningful learning environment for developing higher-order thinking skills. This study contributes to the literature by suggesting that integrating problem-based learning, problem posing, STEM learning, and dynamic visualization within a single instructional model was associated with improved mathematical critical thinking compared with conventional instruction.**

***Keywords*—mathematical critical thinking, propose-STEM learning model, GeoGebra, STEM learning, problem posing, engineering mathematics education**

I. Introduction

In the era of rapid technological advancement and increasingly complex engineering challenges, higher education institutions must equip students with competencies to analyze complex information, evaluate alternative solutions, and make informed decisions in professional contexts. Engineering graduates are expected not only to possess strong technical knowledge but also to demonstrate higher-order thinking skills, particularly analytical reasoning, critical thinking, and problem-solving abilities [1–6]. Among these competencies, mathematical critical thinking is fundamental, as mathematics underpins modeling, optimization, and system analysis in engineering [7–11]. Mathematical critical thinking enables students to interpret quantitative information, analyze relationships among variables, evaluate mathematical arguments, and construct logical conclusions when addressing complex engineering problems [12–15]. Consequently, developing students' mathematical critical thinking has become a central objective in contemporary engineering mathematics education.

However, despite its importance, existing studies consistently report that mathematics instruction in engineering programs remains dominated by procedural teaching and routine problem-solving activities [16–18]. Such instructional practices tend to emphasize the application of formulas and algorithmic procedures rather than fostering conceptual understanding and analytical reasoning. As a result, although students may successfully solve familiar exercises, they often struggle with unfamiliar problems that require interpretation, evaluation, and higher-order reasoning [19, 20]. This mismatch reveals a critical gap between the intended learning outcomes of engineering mathematics and students' actual ability to demonstrate mathematical critical thinking in practice.

To address this gap, various instructional approaches have been proposed in the literature. Problem-Based Learning (PBL), for instance, has been widely recognized for its potential to foster analytical reasoning and collaborative problem-solving skills by engaging students in authentic and context-rich problem situations [21, 22]. Similarly, problem posing has been identified as an effective pedagogical strategy for enhancing students' creativity and deepening conceptual understanding, as it encourages learners to generate, reformulate, and extend mathematical problems [23, 24]. Furthermore, STEM-oriented learning has been shown to promote interdisciplinary thinking and real-world problem-solving abilities by integrating perspectives from science, technology, engineering, and mathematics [25, 26].

While these approaches have demonstrated significant potential, prior studies often report limitations in their

implementation. For instance, PBL may not explicitly foster problem-generation skills, while problem-posing activities are sometimes implemented without sufficient contextual grounding. Similarly, STEM-oriented learning may emphasize interdisciplinary integration but does not always foster deep mathematical reasoning.

Additionally, integrating digital mathematical technologies, such as GeoGebra, offers significant opportunities for dynamic visualization and interactive exploration of mathematical concepts. Prior studies indicate that technology-enhanced learning environments can substantially improve students' conceptual understanding and engagement by enabling them to investigate mathematical relationships through multiple representations, including graphical, symbolic, and numerical forms [27, 28].

The integration of GeoGebra enables dynamic representation and interactive learning experiences that support the development of mathematical critical thinking. In this study, GeoGebra was positioned not merely as a visualization tool but as the central technological component that supported dynamic exploration, multiple representations, and mathematical reasoning throughout the learning process.

Despite these promising findings, existing studies predominantly examine these approaches in isolation rather than within an integrated instructional framework. Examining these approaches separately limits the potential synergy among problem-solving, problem-posing, interdisciplinary learning, and technology-supported exploration. Moreover, many technology-enhanced learning environments emphasize engagement and visualization without explicitly targeting the development of mathematical critical thinking. Consequently, there remains a limited number of empirically validated instructional models that systematically integrate these approaches to support the development of mathematical critical thinking in engineering mathematics education.

To address this gap, this study proposes a GeoGebra-supported Propose-STEM learning model that integrates Problem-Based Learning, problem posing, and STEM-oriented learning within a technology-enhanced environment. The model is designed to create a continuous learning cycle in which students analyze contextual problems, explore mathematical concepts through dynamic visualization, develop solution strategies, and generate new problems based on conceptual understanding.

The originality of Propose-STEM lies in the functional integration and instructional sequencing of four complementary components. Problem-based learning serves as the initial driver for contextual inquiry and analytical engagement. Problem posing extends learning beyond solving given problems by encouraging students to formulate and reconstruct mathematical situations. STEM-oriented learning provides an interdisciplinary engineering context that connects mathematical reasoning with authentic applications. GeoGebra functions as a technological and cognitive mediator that supports dynamic visualization, exploration, and verification of mathematical relationships.

These components are operationalized through five instructional phases: (1) Problem Orientation, (2) Evaluation and Investigation, (3) Analysis and Problem Solving, (4) Repose and Problem Posing, and (5) Learning Reflection. Unlike instructional approaches that primarily focus on solving predefined problems or supporting conceptual exploration as separate activities, Propose-STEM establishes a recursive learning process in which students move iteratively between contextual problem solving, mathematical problem generation, evaluation of reasoning, and technology-supported conceptual refinement. Through this sequence, students engage in inquiry, exploration, and reflective reasoning to deepen conceptual understanding and support the development of mathematical critical thinking.

Accordingly, this study contributes to the literature in three ways. First, it proposes a theoretically grounded instructional model that systematically integrates PBL, problem posing, STEM-oriented learning, and GeoGebra within a unified pedagogical structure rather than implementing these approaches as isolated instructional strategies. Second, it conceptualizes GeoGebra as an embedded cognitive component that supports investigation, verification, and reflective reasoning, rather than merely as a visualization tool. Third, it provides empirical evidence regarding how this integrated instructional process is associated with the development of mathematical critical thinking in engineering mathematics.

Therefore, this study aims to investigate how integrating GeoGebra into the Propose-STEM learning model is associated with students' mathematical critical thinking in engineering mathematics.

To provide clearer direction for this study, the following research questions and hypotheses are formulated:

RQ1: Does the GeoGebra-supported Propose-STEM learning model significantly improve students' mathematical critical thinking compared to conventional instruction?

H1: Students taught using the Propose-STEM learning model supported by GeoGebra will demonstrate significantly higher mathematical critical thinking skills than students taught using conventional teaching methods.

II. Literature Review

A. Mathematical Critical Thinking

Mathematical critical thinking is generally regarded as a core competency in higher education, particularly in disciplines that rely heavily on quantitative reasoning, such as engineering. In contemporary learning environments, students are expected not only to perform mathematical computations but also to analyze complex information, evaluate alternative solution strategies, and construct logical arguments when solving unfamiliar problems. Mathematical critical thinking involves the ability to interpret mathematical situations, analyze relationships among variables, assess the validity of reasoning, and justify conclusions based on sound mathematical principles [29–32]. These cognitive abilities enable students to approach mathematical problems analytically and make informed decisions when dealing with complex quantitative information.

In engineering education, mathematical critical thinking skills are particularly essential for modeling systems, analyzing data, and optimizing engineering solutions [11]. However, several studies indicate that students often experience difficulties in developing mathematical critical thinking due to instructional practices that emphasize

procedural fluency and routine problem-solving rather than analytical reasoning [33–35]. Such practices may limit students' opportunities to explore mathematical structures and critically evaluate solution strategies.

The conceptualization of critical thinking in this study is grounded in the framework proposed by Facione [36], which includes core dimensions such as interpretation, analysis, evaluation, inference, explanation, and self-regulation. However, in the context of mathematics education, several studies suggest that only specific dimensions directly reflect reasoning processes in problem-solving. Interpretation, analysis, evaluation, and inference are considered the most relevant cognitive processes for understanding mathematical problems, examining relationships among variables, assessing solution strategies, and drawing logical conclusions. Therefore, these four indicators are adopted in this study to capture the essential aspects of mathematical critical thinking in quantitative learning contexts.

Traditional teaching approaches tend to focus on applying formulas and algorithmic procedures rather than encouraging students to explore mathematical structures or critically evaluate solution strategies [37, 38]. As a result, students may successfully solve routine exercises but struggle when confronted with complex or unfamiliar problems that require interpretation, reasoning, and evaluation.

Recent research further highlights the importance of designing learning environments that actively engage students in inquiry, discussion, and reflective thinking to foster mathematical critical thinking. Learning approaches that involve authentic problems, collaborative exploration, and opportunities for students to analyze and justify their reasoning have been shown to support deeper conceptual understanding and development of critical thinking [39–41]. Such environments encourage students to examine mathematical relationships from multiple perspectives and develop the ability to evaluate mathematical arguments critically.

However, there remains a need for instructional approaches that systematically integrate these elements to effectively support the development of mathematical critical thinking in engineering mathematics education.

B. Problem-based Learning in Engineering Mathematics

Problem-Based Learning (PBL) is extensively regarded as an effective instructional approach for promoting higher-order thinking skills in higher education. It emphasizes student-centered learning in which students actively engage in solving authentic, complex problems rather than passively receiving information from instructors. Through problem-centered learning environments, students investigate contextual situations, identify relevant information, and collaboratively develop solution strategies. Such environments support deeper conceptual understanding, as students are required to interpret problem contexts, analyze relationships among variables, and justify their reasoning using mathematical arguments [42–44].

In engineering mathematics education, PBL is particularly relevant because engineering practice inherently involves addressing complex and ill-structured problems that require analytical reasoning and interdisciplinary knowledge. Students are expected to apply mathematical concepts to model systems, analyze technical data, and evaluate alternative solutions. Accordingly, PBL-based learning environments enable students to connect theoretical mathematical concepts with real-world engineering contexts while fostering analytical and decision-making skills [17, 45]. Empirical studies further suggest that PBL can enhance conceptual understanding and critical thinking by encouraging students to explore multiple solution strategies and justify their reasoning through collaborative discussion [32, 46].

The integration of digital learning environments further strengthens the potential of PBL. Technology-enhanced learning platforms can facilitate collaborative inquiry, reflective thinking, and deeper engagement in problem-solving activities. When students interact with authentic problems in technology-supported environments, they are more likely to analyze mathematical relationships, evaluate alternative solutions, and develop more sophisticated reasoning processes [47, 48].

Although PBL primarily supports analytical reasoning and collaborative problem-solving, it primarily emphasizes solving instructor-provided problems and may offer limited opportunities for students to generate their own mathematical problems. As a result, students may not fully develop the ability to explore alternative problem situations or examine mathematical structures in depth. This limitation suggests that PBL alone may be insufficient to develop comprehensive mathematical critical thinking skills. Therefore, integrating PBL with complementary approaches, such as problem posing and technology-supported exploration, may provide a more comprehensive learning environment that fosters both problem-solving and problem-formulation abilities in mathematics learning.

C. Problem Posing in Mathematics Learning

Problem posing is considered a powerful pedagogical strategy in mathematics education for fostering deeper conceptual understanding and higher-order thinking skills. It involves students in formulating, modifying, or extending mathematical problems based on given contexts [49–51]. Through problem-posing activities, students explore mathematical relationships, analyze problem structures, and reflect on the reasoning processes underlying solution construction. This active engagement enables students to move beyond routine procedures and develop a more flexible understanding of mathematical concepts.

Empirical studies reveal that engaging students in problem posing can significantly enhance mathematical reasoning and critical thinking. When students construct their own problems, they are required to analyze relationships among variables, evaluate possible conditions, and consider multiple solution strategies, thereby promoting flexible thinking and deeper exploration of mathematical ideas [24, 52]. In addition, problem-posing activities have been shown to support creativity, reflective thinking, and stronger connections across different mathematical representations [53, 54].

However, problem posing is often implemented independently from structured problem-solving approaches, which may limit its effectiveness in supporting comprehensive mathematical understanding. Without sufficient instructional guidance, students may experience difficulties in generating meaningful problems or in linking

problem-posing activities to broader learning objectives [55, 56]. This limitation highlights the need to integrate problem-posing with structured pedagogical approaches and technology-supported environments to create more coherent and effective learning experiences.

D. STEM-oriented Learning in Engineering Education

STEM (Science, Technology, Engineering, and Mathematics) education has been recognized as an effective approach for preparing students to address complex challenges in modern technological societies. STEM-oriented learning emphasizes integrating knowledge and practices from multiple disciplines, enabling students to solve real-world problems through inquiry, design, and analytical reasoning. In such environments, students apply scientific concepts, technological tools, engineering design processes, and mathematical reasoning in a coordinated manner when analyzing complex phenomena [57–59]. This interdisciplinary approach aims to develop essential twenty-first-century competencies, including critical thinking, problem solving, collaboration, and creativity.

In engineering education, STEM-oriented learning plays a particularly important role because engineering problems inherently require integrating knowledge from multiple domains. Students are expected to interpret data, analyze relationships among variables, and evaluate alternative solutions, drawing on both theoretical understanding and practical considerations. STEM learning environments, therefore, provide opportunities for design-based problem solving and interdisciplinary exploration, enabling students to develop a deeper understanding of how mathematical concepts are applied in engineering contexts [60, 61].

Empirical studies imply that STEM-oriented instructional approaches can enhance students' higher-order thinking skills, particularly when learning activities involve inquiry, experimentation, and design-based problem-solving. In such environments, students actively explore complex problems, construct mathematical models, and evaluate alternative solutions using interdisciplinary knowledge. This process supports the development of critical thinking by encouraging students to analyze problems from multiple perspectives and justify their reasoning through evidence-based arguments [9].

Furthermore, STEM learning environments frequently incorporate digital technologies that support the exploration and visualization of complex phenomena. Technology-enhanced environments enable students to simulate real-world scenarios, analyze mathematical relationships, and refine their reasoning processes through interactive experiences. Such integration has been shown to facilitate deeper engagement and strengthen the connection between theoretical knowledge and practical problem solving [35].

Nevertheless, STEM-based activities offer several benefits and are often implemented without explicitly targeting the development of mathematical reasoning and critical thinking skills. Therefore, students may engage in interdisciplinary tasks without fully developing the analytical competencies required in engineering practice. This limitation highlights the need for instructional models that systematically integrate STEM learning principles with structured problem-solving processes and technology-supported exploration to foster effective mathematical critical thinking.

E. GeoGebra in Technology-enhanced Mathematics Learning

The integration of digital technologies has become increasingly central in contemporary mathematics education, particularly in higher education, where students are expected to develop analytical and problem-solving skills in technology-rich environments. Technology-enhanced learning environments let students explore mathematical concepts dynamically, visualize abstract relationships, and engage in interactive experimentation. Such environments support deeper conceptual understanding by allowing learners to manipulate mathematical objects, test conjectures, and observe the effects of parameter changes in real time [62].

One commonly used digital tool in mathematics education is GeoGebra, an interactive dynamic mathematics software that integrates multiple mathematical representations, including algebraic expressions, graphical visualizations, and numerical models. GeoGebra enables students to construct mathematical objects, explore relationships among variables, and investigate mathematical patterns through dynamic manipulation. These capabilities make GeoGebra particularly useful for supporting exploratory learning and conceptual understanding in mathematics [30].

Empirical studies show that using GeoGebra can enhance students' conceptual understanding, mathematical reasoning, and engagement in learning activities. Through dynamic visualization, students can explore relationships that are difficult to observe using static representations, experiment with mathematical models, and evaluate alternative solution strategies. These interactive processes promote deeper cognitive engagement and support the development of analytical thinking in mathematics learning [27, 63–65].

In engineering mathematics education, GeoGebra offers valuable opportunities to visualize mathematical models commonly used in engineering analysis, such as functions, derivatives, and optimization problems. Interactive visualization tools help students understand how mathematical relationships influence engineering systems and how mathematical models represent real-world phenomena [66–68]. Through such dynamic exploration, students can investigate complex relationships and evaluate problem-solving strategies more effectively.

Although digital tools such as GeoGebra offer several advantages, their effectiveness depends largely on how they are integrated within pedagogical frameworks. Technology alone does not automatically enhance learning outcomes unless it is embedded within instructional approaches that promote inquiry, problem-solving, and critical analysis. Therefore, integrating GeoGebra with pedagogical approaches such as problem-based learning, problem posing, and STEM-oriented learning is necessary to create more coherent learning environments that effectively support the development of mathematical critical thinking in engineering mathematics education.

F. Theoretical Foundation and Instructional Integration of the Propose-STEM Model

Although problem-based learning, problem posing, STEM-oriented learning, and GeoGebra-supported

instruction have each demonstrated potential for improving higher-order thinking and mathematical learning outcomes, previous studies have generally examined these approaches independently. As a result, existing instructional practices tend to emphasize particular dimensions of learning without fully supporting the interconnected processes involved in developing mathematical critical thinking.

The Propose-STEM learning model was developed to address this limitation by integrating theoretical frameworks into an instructional structure. Rather than introducing entirely new pedagogical components, the model combines established instructional approaches according to complementary instructional functions and organizes them into a coherent learning sequence.

Within this framework, problem-based learning functions as the entry point for contextual inquiry, engaging students in authentic engineering situations that require interpretation and analytical reasoning. Problem posing extends this process by encouraging students to reformulate assumptions, construct alternative mathematical situations, and reflect on relationships among variables. STEM-oriented learning situates mathematical reasoning within interdisciplinary engineering contexts and supports connections between conceptual understanding and real-world applications. GeoGebra serves as a technological and cognitive mediator that supports dynamic visualization, conceptual exploration, and verification throughout the learning process. To clarify how these components are theoretically integrated and operationalized within the Propose-STEM framework, Table 1 summarizes their theoretical basis, instructional function, and primary contribution to the development of mathematical critical thinking.

Table 1. Theoretical integration and instructional functions of the Propose-STEM learning model

Component	Primary Theoretical Basis	Instructional Function within Propose-STEM	Primary Contribution to Mathematical Critical Thinking
Problem-Based Learning (PBL)	Inquiry-based and contextual learning	Initiates contextual inquiry and supports analytical engagement through authentic engineering problems	Interpretation and Analysis
Problem Posing	Reflective and generative learning	Encourages students to reformulate assumptions and generate alternative mathematical situations	Evaluation and Inference
STEM-oriented Learning	Interdisciplinary and situated learning	Connect mathematical reasoning with authentic engineering applications	Analysis and Evaluation
GeoGebra	Technology-enhanced learning and multiple representations	Supports dynamic visualization, exploration, and verification of mathematical relationships	Interpretation, Analysis, and Inference
Integrated Propose–STEM	Sequential instructional integration	Establishes recursive interaction among inquiry, exploration, problem reformulation, and reflection	Development of Mathematical Critical Thinking

As summarized in Table 1, the Propose-STEM model does not treat these instructional components as independent learning strategies. Instead, each component serves a complementary function and contributes to distinct dimensions of mathematical critical thinking through a structured, interconnected instructional process.

The integration of these components is grounded in the premise that mathematical critical thinking develops through recursive interactions among interpretation, exploration, evaluation, problem reformulation, and reflective reasoning. Accordingly, Propose-STEM organizes learning into five interconnected phases: problem orientation, evaluation and investigation, analysis and problem solving, repose and problem posing, and learning reflection.

This structure differentiates Propose-STEM from instructional implementations that primarily emphasize solving predefined problems or treating conceptual exploration as separate activity. Instead, the model establishes a recursive learning process in which students iteratively move between contextual problem solving, mathematical problem generation, evaluation of reasoning, and technology-supported conceptual refinement.

Based on this theoretical integration, Propose–STEM is expected to provide a more coherent environment for supporting the development of critical mathematical thinking in engineering mathematics.

III. Materials and Methods

A. Research Design

This study employed a quasi-experimental research design to investigate the effectiveness of the GeoGebra-supported Propose-STEM learning model in enhancing students' mathematical critical thinking in engineering mathematics courses. A quasi-experimental design (Fig. 1) was selected because the participants were drawn from intact classes in an authentic educational setting, where random assignment of individual students was not feasible [69].

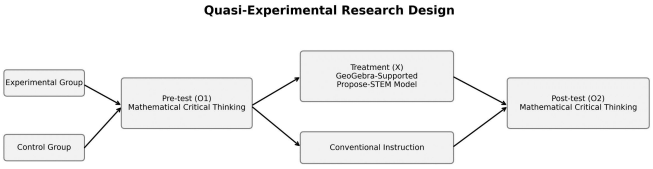


Fig. 1. Research design.

Specifically, the study employed a non-equivalent control group pretest-posttest design, comprising an experimental group and a control group. Such designs are widely used in educational research to evaluate instructional interventions in natural classroom settings. The experimental group was taught by using the GeoGebra-supported Propose-STEM learning model, while the control group was taught using conventional instructional methods.

Both groups completed a pretest before the instructional intervention and a posttest after the learning activities. The pretest was administered to measure students' initial level of mathematical critical thinking and to ensure comparability between the two groups before the intervention. Following the instructional treatment, a posttest was administered to assess students' mathematical critical thinking.

B. Participants

The participants in this study were 73 undergraduate students from the Civil Engineering program at Universitas Negeri Padang who were enrolled in an engineering mathematics (calculus) course during the 2025/2026 diverse

academic abilities and learning characteristics.

Participants were selected using a purposive sampling technique, a commonly employed approach in educational research that selects participants who meet specific research criteria [70, 71]. Two intact classes were involved in the study: one class served as the experimental group and received instruction using the GeoGebra-supported Propose-STEM learning model. In contrast, the other class served as the control group and received conventional instruction. Both groups studied the same calculus topics during the instructional period to ensure consistency in learning content.

C. Research Instruments

The primary instrument used in this study was a mathematical critical thinking test consisting of five open-ended items. The instrument was developed to assess four indicators of mathematical critical thinking: interpretation, analysis, evaluation, and inference. The indicators were operationalized based on students' ability to identify mathematical information, examine relationships among concepts, justify solution strategies, and formulate logical conclusions in engineering mathematics contexts.

Although the Propose-STEM learning model conceptually incorporates explanation as an element of mathematical critical thinking, explanation was not treated as an independent indicator in this study. Instead, explanation was conceptually embedded within the evaluation and inference indicators, particularly through students' ability to justify reasoning processes and articulate mathematically supported conclusions.

The scoring rubric was developed based on these indicators, as presented in Table 2, to ensure alignment between the measurement construct and the assessment criteria.

Each indicator was scored on a scale of 0 to 4, yielding a maximum score of 16 for each item. Since the instrument consisted of five items, the maximum possible raw score was 80. The total score was calculated by summing all item scores and then converting the raw score to a standardized score on a scale of 0–100.

The instrument employed essay-type questions to allow students to demonstrate reasoning processes, argument construction, and mathematical justification during problem solving. Each item was intentionally designed to elicit multiple dimensions of mathematical critical thinking rather than procedural computation alone. Examples of assessment items, sample student responses, and illustrations of rubric application are provided in Appendix A to demonstrate how interpretation, analysis, evaluation, and inference were operationalized during scoring.

Table 2. Scoring rubric for mathematical critical thinking

Score	Interpretation	Analysis	Evaluation	Inference
4 (Excellent)	Accurately and completely identifies all relevant information and clearly interprets the problem	Correctly analyzes relationships among variables with logical and systematic reasoning	Critically evaluates solution strategies and justifies decisions with strong mathematical arguments	Draws correct and well-supported conclusions with clear justification
3 (Good)	Identifies most relevant information and demonstrates a generally correct interpretation	Analyzes relationships with minor errors or incomplete reasoning	Evaluates solution strategies with partial justification	Draws mostly correct conclusions with some justification
2 (Fair)	Identifies some relevant information but interpretation is partially incorrect	Limited analysis with noticeable errors in reasoning	Provides minimal evaluation with weak justification	Draws partially correct conclusions with limited reasoning
1 (Poor)	Misinterprets the problem or identifies very limited relevant information	Incomplete or incorrect analysis	Lacks evaluation or justification	Draws incorrect or unsupported conclusions
0 (No response)	No response or irrelevant answer	No response or irrelevant answer	No response or irrelevant answer	No response or irrelevant answer

Table 3. Inter-rater reliability analysis using ICC

Variable assessed	ICC value
Mathematical critical thinking test score	0.71

Table 4. Indicators of mathematical critical thinking

Indicator	Description
Interpretation	Ability to understand and interpret mathematical problems and identify relevant information.
Analysis	Ability to analyze relationships among variables and organize information to solve problems.
Evaluation	Ability to assess the validity of solution strategies and mathematical arguments.
Inference	Ability to draw logical conclusions based on mathematical reasoning.

Student responses were independently evaluated by two raters using the same analytic rubric. Inter-rater reliability was assessed using the Intraclass Correlation Coefficient (ICC). Specifically, a two-way random-effects model with absolute agreement [ICC(2,k)] was employed because both raters independently scored the same responses, and agreement between raters was the primary concern. As shown in Table 3, the ICC of 0.71 indicates good inter-rater reliability and supports consistency in the scoring procedure.

The indicators used to measure students' mathematical critical thinking are presented in Table 4.

The instrument consisted of essay-type questions, allowing students to explain their reasoning processes in solving mathematical problems. Each response was evaluated using a scoring rubric aligned with the four indicators of mathematical critical thinking. Content validity was established through expert judgment from five reviewers: two experts in mathematics education, two in engineering mathematics, and one in educational evaluation. The reviewers evaluated each item based on its relevance to the construct, wording clarity, cognitive appropriateness, and alignment with the intended mathematical critical thinking indicators. Recommendations from the experts were incorporated to refine item formulation, scoring descriptors, and instructional alignment prior to implementation.

D. Instrument Validity and Reliability

1) Instrument validity

The instrument's validity was evaluated using expert judgment to ensure that the test items adequately represented

the intended critical-thinking indicators. The level of agreement among the experts was analyzed using Aiken's V coefficient to evaluate content validity. The average Aiken's V coefficient from expert validation was 0.934, indicating high content validity.

2) Instrument reliability

The instrument's reliability was evaluated using Cronbach's Alpha coefficient to assess its internal consistency [72]. The analysis yielded Cronbach's alpha of 0.861, indicating that the instruments demonstrated acceptable reliability in measuring students' mathematical critical thinking abilities.

E. Data Collection Procedure

The data collection procedure was carried out in several stages. First, a pretest was administered to the experimental and control groups to assess students' initial level of mathematical critical thinking and to ensure comparability between the groups before the instructional intervention.

The instructional treatment was implemented over eight weeks, consisting of eight sessions, with each session equivalent to three credit hours (150 min). The learning activities covered applications of differential calculus, including maximum and minimum values, monotonicity and concavity, local maxima and local minima, and the use of derivatives in problem-solving contexts.

The experimental group participated in the GeoGebra-supported Propose-STEM learning model, which integrates problem-based learning, problem posing, STEM-oriented learning, and GeoGebra-assisted exploration. In contrast, the control group received conventional instruction consisting of lecturer-centered explanations, demonstration of solution procedures, and routine exercises without GeoGebra or inquiry-oriented activities.

To minimize instructor-related variability, both groups were taught by the same instructor. Treatment fidelity was maintained through the consistent implementation of structured lesson plans, learning objectives, and assessment criteria across all sessions.

Following the intervention, a posttest was administered to assess students' mathematical critical thinking after the learning activities.

F. Instructional Framework: GeoGebra-supported Propose-STEM Learning Model

Building upon the theoretical foundations of Problem-Based Learning, problem posing, STEM-oriented learning, and technology-enhanced mathematics instruction, this study employs a GeoGebra-supported Propose-STEM learning model to enhance students' mathematical critical thinking in engineering mathematics education. Previous studies suggest that integrating inquiry-based pedagogy with digital technologies can create more effective environments for fostering higher-order thinking and conceptual understanding [73–75].

The Propose-STEM learning model combines problem-based learning, problem posing, interdisciplinary STEM perspectives, and GeoGebra-assisted visualization within a unified instructional process. In this framework, students engage in contextual engineering problems, generate and extend mathematical problems, and explore mathematical relationships through dynamic visualization and interactive exploration.

GeoGebra was systematically embedded throughout each instructional phase to support visualization, exploration, verification, and problem-solving, ensuring it served as a central component of the instructional design rather than merely as a supplementary tool.

Through this integrated approach, students engage in inquiry, collaboration, reflection, and mathematical formulation, thereby supporting the development of interpretation, analysis, evaluation, and inference as essential components of mathematical critical thinking. In this study, explanation was not treated as an independent dimension. Still, it was conceptually embedded within the evaluation and inference indicators, particularly through students' ability to justify reasoning, and articulate logical conclusions. The conceptual relationship among the learning components, instructional processes, and mathematical critical thinking indicators is illustrated in Fig. 2.

Based on the conceptual framework, the instructional phases of the GeoGebra-supported Propose-STEM learning model were translated into a structured instructional sequence consisting of five interconnected phases. Fig. 3 illustrates the recursive learning cycle, while Table 5 presents the operational instructional syntax and the role of each instructional component across the learning process.

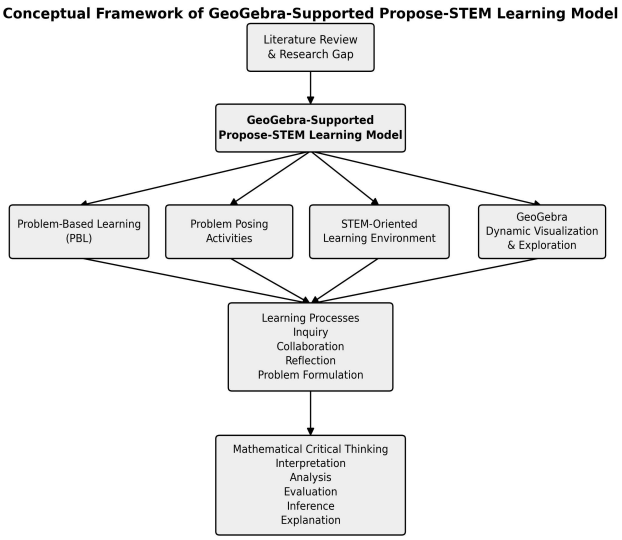


Fig. 2. The conceptual framework of the propose-STEM learning model.

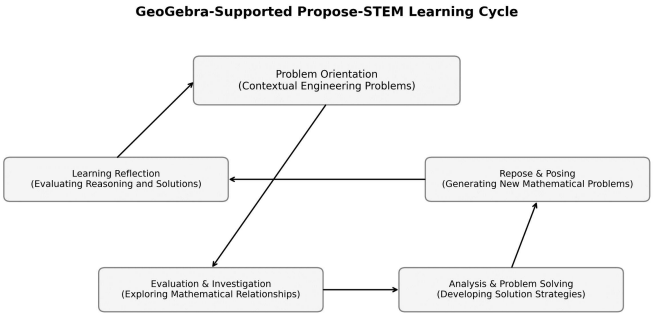


Fig. 3. Instructional phase.

As shown in Table 5, each phase performs a distinct instructional function while maintaining coherence across contextual inquiry, conceptual exploration, problem

reformulation, and reflective reasoning. GeoGebra was systematically embedded throughout the instructional process to support dynamic exploration and mathematical reasoning rather than functioning solely as a visualization tool.

Table 5. Instructional syntax of the GeoGebra-supported propose-STEM learning model

Phase	Dominant Components	Instructional Function	Lecturer Role	Student Activities	GeoGebra Integration
Problem Orientation	Problem-Based Learning (PBL)	Initiate contextual inquiry and identify mathematical problems within engineering contexts	Introduce contextual engineering problems.	Identify key information and interpret the problem.	Visualize mathematical representations.
Evaluation & Investigation	GeoGebra + STEM-oriented Learning	Explore and evaluate relationships among mathematical concepts through dynamic investigation	Facilitate inquiry and concept exploration.	Investigate variables and discuss solution approaches.	Explore graphs and relationships among variables.
Analysis & Problem Solving	STEM-oriented Learning	Develop analytical reasoning and collaborative problem-solving strategies	Guide problem-solving and provide feedback.	Develop solution strategies collaboratively.	Model and verify solutions dynamically.
Repose & posing	Problem Posing	Extend reasoning through mathematical reformulation and problem generation	Encourage students to reformulate and extend problems.	Generate new mathematical problems based on given situations	Modify parameters to explore different scenarios.
Learning Reflection	Integrated Components	Promote reflective reasoning and consolidate conceptual understanding	Facilitate reflection and summarize key concepts.	Evaluate solutions and reflect on reasoning processes.	Confirm results through visualization.

G. Data Analysis

The collected data were analyzed using both descriptive and inferential statistical methods. Descriptive statistics were used to summarize students' pretest and posttest scores in both the experimental and control groups.

To examine improvements in students' mathematical critical thinking, the normalized gain (N-gain) was calculated to quantify of learning gains after the instructional intervention. The N-gain value was calculated using the following formula [76]:

$$g = \frac{\text{Posttest} - \text{Pretest}}{\text{Maximum Score} - \text{Pretest}} \quad (1)$$

The interpretation of the N-gain values is presented in Table 6.

Table 6. Normalized gain classification

N-gain Value	Category
$g \geq 0.70$	High
$0.30 \leq g < 0.70$	Medium
$g < 0.30$	Low

Before conducting the independent-samples t-test, the assumptions of normality and homogeneity of variance were examined to ensure the data met the requirements for parametric statistical analysis.

An independent samples t-test was conducted to examine differences in posttest scores between the experimental and control groups after the learning model was implemented. In addition, an Analysis of Covariance (ANCOVA) was performed using posttest scores as the dependent variable, instructional group as the independent variable, and pretest scores as the covariate to control for potential baseline differences between groups. The statistical analysis was performed at the 0.05 significance level.

Effect size (Cohen's d) was also calculated to determine the magnitude of the difference between the two groups and to provide information regarding the practical significance of the intervention. The effect size was calculated using the following formula:

$$d = \frac{M_1 - M_2}{SD_{pooled}} \quad (2)$$

The interpretation of the effect size values is presented in Table 7 [77].

Table 7. Effect size interpretation

Effect Size (d)	Interpretation
$d \leq 0.20$	Small
$0.50 \leq d < 0.80$	Medium
$d \geq 0.80$	Large

H. Implementation of the GeoGebra-supported Propose-STEM Learning Model

The GeoGebra-supported Propose-STEM learning model was implemented through a sequence of instructional phases designed to promote inquiry, collaborative problem solving, and technology-enhanced mathematical exploration. GeoGebra served as a dynamic mathematical environment that enabled students to explore mathematical concepts through interactive visualization and multiple representations. The integration of GeoGebra enabled students to simultaneously observe algebraic expressions, derivative functions, and graphical representations, thereby supporting deeper conceptual understanding of calculus.

To improve transparency in implementation and support replication, the instructional intervention was organized into eight sessions, with structured lesson plans and standardized learning worksheets. Each session employed contextual engineering mathematics tasks related to civil engineering applications of differential calculus, including roadway embankment optimization, structural profile analysis, material efficiency, water channel design, and derivative-based engineering modeling. Although the instructional phases remained consistent across sessions, the contextual problems and mathematical tasks varied to encourage broader application of mathematical critical thinking.

During the evaluation and investigation phase, GeoGebra was actively integrated into learning activities to support students in constructing and manipulating dynamic mathematical representations. Students explored the mathematical structure of functions, including their first and second derivatives, and examined how the sign of the second derivative influences the curvature of the graph. Through this

exploration, students identified concave-upward and concave-downward intervals while analyzing the relationship between symbolic differentiation and graphical interpretation. These activities positioned GeoGebra as a central technological component of the learning process by facilitating interactive exploration and mathematical reasoning. Figs. 4 and 5 illustrate examples of GeoGebra-supported explorations used during the learning process.

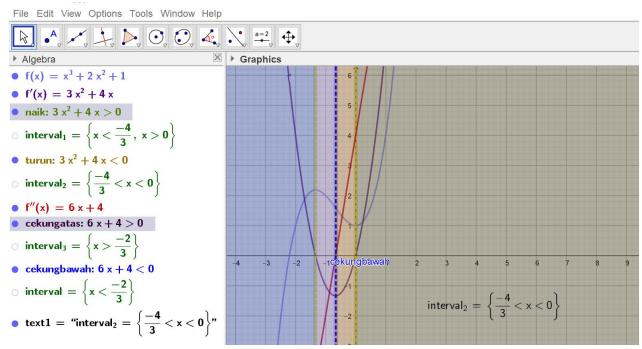


Fig. 4. Student exploration using GeoGebra Classic 5 to analyze concave upward and concave downward intervals of a function through the visualization of the first and second derivatives.

Fig. 6 illustrates how GeoGebra displays functions together with their derivatives, allowing students to investigate concavity and identify inflection points through direct graphical observation.

These activities encouraged students to interpret graphical information, analyze connections between derivatives and the behavior of functions, evaluate solution strategies, and formulate conclusions based on visual and symbolic evidence. Through dynamic manipulation and immediate visual feedback, students tested conjectures, refined their reasoning, and developed key dimensions of mathematical critical thinking, including interpretation, analysis, evaluation, and inference.

In the analysis and problem-solving phase, students collaboratively interpreted their findings and justified their solutions by connecting derivative values with graphical behavior. The problem-posing phase encouraged students to generate new mathematical problems by modifying functions and exploring alternative scenarios.

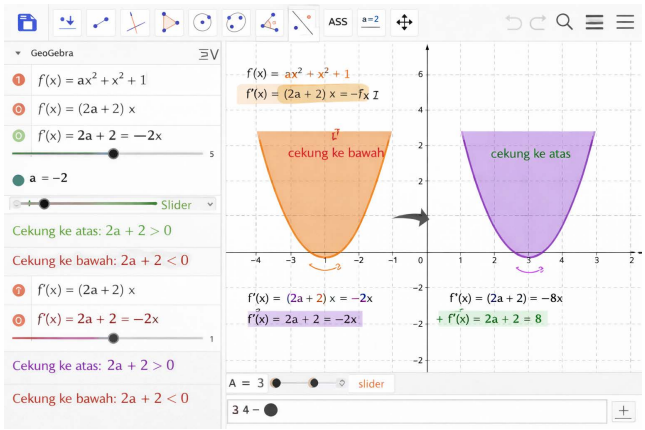


Fig. 5. Dynamic exploration of concave upward and concave downward behavior of a function using GeoGebra 6. Students manipulated parameters and analyzed the first and second derivatives to understand how the sign of the second derivative determines the curvature of the function.

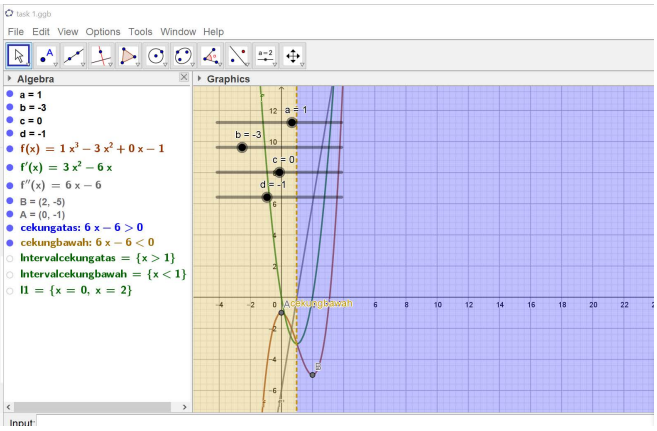


Fig. 6. Student exploration of function concavity using GeoGebra 5. Students manipulated parameters of the function $f(x) = ax^3 + bx^2 + cx + d$using sliders to investigate the relationship between the function, its first derivative, and second derivative, and to determine intervals of concave upward and concave downward behavior.

Table 8. Summary of instructional activities during implementation

Instructional phase	Engineering of engineering problem	Activities conducted	Role of GeoGebra	Expected Mathematical critical thinking indicators
Problem Orientation	In a highway construction project, the elevation profile of an embankment is modeled by ($h(x) = x^3$-$3x^2$-1), where (x) represents horizontal distance (m). Determine whether the profile satisfies engineering requirements for safe elevation changes.	Students interpreted the engineering problem, identified known information, formulated assumptions, and represented the problem mathematically.	Visualize the initial mathematical model and contextual situation.	Interpretation
Evaluation & Investigation	Same problem continued. Students investigated first and second derivatives to determine slope changes, concavity, and possible inflection points	Students solved the problem collaboratively, justified procedures, and interpreted mathematical findings.	Manipulate parameters dynamically and observe graphical changes	Interpretation, Analysis
Analysis & Problem Solving	Same problem continued. Students determined feasible engineering decisions based on derivative analysis and graphical interpretation.	Students modified assumptions, generated alternative problems, and proposed new solution scenarios.	Verify symbolic solutions using graphical exploration.	Analysis, Evaluation
Repose & posing	Same problem continued. Students generated alternative embankment models by modifying coefficients and engineering constraints	Students modified assumptions, generated alternative problems, and proposed new solution scenarios.	Test modified functions and compare outcomes dynamically	Evaluation, Inference
Learning Reflection	Same problem continued. Students evaluated the proposed solutions and reflected on relationships among mathematical representations and engineering implications.	Students reflected on conclusions and evaluated relationships among mathematical representations.	Review graphical evidence and summarize conceptual understanding.	Inference

During the Repose and Posing phase, students were instructed to generate new mathematical problems by modifying assumptions, changing parameter values, extending engineering constraints, or proposing alternative scenarios derived from the original task. Students worked collaboratively to evaluate whether the generated problems remained mathematically and contextually feasible and justified possible solution approaches before presenting their ideas to peers.

To provide a clearer description of how the instructional intervention was conducted and to support replicability, examples of instructional activities implemented across the phases of the GeoGebra-supported Propose-STEM learning model are summarized in Table 8. The table presents representative engineering contexts, GeoGebra-supported exploration activities, and problem-posing instructions employed during the learning process.

Finally, during the reflection phase, students evaluated relationships among algebraic expressions, derivatives, and graphical representations to consolidate their conceptual understanding.

Overall, integrating GeoGebra into the Propose-STEM learning model created a technology-enhanced learning environment that supported inquiry, visualization, and higher-order mathematical reasoning in engineering mathematics contexts.

I. Ethical Approval and Informed Consent

This study comprised ethical standards for research involving human participants. Before the study was implemented, ethical approval was obtained from the institutional research ethics committee of the participating institution.

All participants were informed of the objectives and procedures of the study before data collection began. Participation in the study was voluntary, and students were assured that the data collected would be used solely for research purposes. Written informed consent was obtained from all participants before their involvement in the study.

In addition, all collected data were anonymized to protect participants' identities and confidentiality. No personally identifiable information was included in the research data or reported in the publication.

IV. Results

A. Descriptive Statistics of Mathematical Critical Thinking

This section presents the descriptive statistics of students' mathematical critical thinking scores before and after the instructional intervention. The statistics include the number of participants (N), mean score, standard deviation, and range of scores for both the experimental and control groups.

Table 9 shows the descriptive statistics of students' mathematical critical thinking scores before and after the intervention. The experimental group obtained a higher observed posttest mean (M = 82.43, SD = 8.38) than the control group (M = 68.75, SD = 10.10), indicating a notable difference in post-intervention performance. However, interpretation of these descriptive differences should be approached with caution because preliminary analysis showed that the groups were not fully equivalent at baseline.

Table 9. Descriptive statistics of mathematical critical thinking scores

Group	N	Pretest mean (SD)	Posttest mean (SD)	Mean difference	Min	Max
Experimental (GeoGebra-supported Propose-STEM)	37	41.35 (8.55)	82.43 (8.38)	41.08	55	95
Control (Conventional Instruction)	36	35.28 (10.48)	68.75 (10.10)	33.47	55	75

The pretest results indicated that the experimental group had a higher initial mean score (M = 41.35, SD = 8.55) than the control group (M = 35.28, SD = 10.48), suggesting initial differences in mathematical critical thinking performance between groups. Therefore, Analysis of Covariance (ANCOVA) was conducted using pretest scores as a covariate to estimate adjusted posttest differences after controlling for baseline performance.

To further examine changes in students' mathematical critical thinking, a normalized gain (N-gain) analysis was conducted to describe the magnitude of improvement within each group across the intervention period.

B. Pretest Equivalence Analysis and ANCOVA

An independent samples t-test was conducted to examine whether there was a significant difference in pretest scores between the experimental and control groups. The results showed a statistically significant difference between the two groups ($t = 2.789$, $p = 0.008$), suggesting that the groups were not equivalent at baseline.

Given this initial difference, an Analysis of Covariance (ANCOVA) was employed using pretest scores as a covariate to adjust posttest outcomes and estimate group differences after controlling for baseline performance.

Before conducting ANCOVA, assumption testing was performed. The assumptions of normality and homogeneity of variances were satisfied based on the Shapiro–Wilk and Levene's test results. In addition, homogeneity of regression slopes was examined by testing the interaction between group and pretest scores. The interaction effect was not statistically significant ($p > 0.05$), indicating that the relationship between pretest and posttest scores was comparable across groups and supporting the use of ANCOVA.

The ANCOVA results revealed that the covariate (pretest) had a significant effect on posttest scores ($F(1, 70) = 18.27$, $p < 0.001$, partial $\eta^2 = 0.207$), indicating that students' initial mathematical critical thinking performance was associated with post-intervention outcomes.

More importantly, after controlling for pretest differences, the effect of the instructional model remained statistically significant ($F(1, 70) = 25.82$, $p < 0.001$, partial $\eta^2 = 0.270$), indicating differences in adjusted posttest mathematical critical thinking scores between groups. Detailed ANCOVA results are presented in Table 10.

These findings suggest that participation in the GeoGebra-supported Propose–STEM learning model was associated with higher outcomes in mathematical critical thinking compared with conventional instruction, after accounting for baseline performance.

To provide a clearer interpretation of group differences after controlling for baseline performance, adjusted posttest

means were estimated through ANCOVA. The adjusted means remained higher in the experimental group than in the control group after controlling for baseline differences, indicating that the observed group differences persisted. The adjusted posttest means are presented in Table 11.

Table 10. ANCOVA results for posttest mathematical critical thinking scores

Source	Sum of square	df	F	Sig. p	Partial η^2
Pretest (Covariate)	1321.84	1	18.27	<0.001	0.207
Instructional group	1867.61	1	25.82	<0.001	0.270
Error	5064.11	70			
Corrected total	8253.56	72			

Table 11. Adjusted posttest means based on ANCOVA

Group	Raw mean	Adjusted Mean	Standard error	95% Confidence Interval
Experimental	82.43	81.12	1.25	78.29–83.63
Control	68.75	70.10	1.79	67.55–72.97

Table 11 presents the adjusted posttest means estimated through ANCOVA, controlling for baseline differences. Although adjustment reduced the observed mean difference between groups, the experimental group continued to show higher performance in mathematical critical thinking than the control group.

C. Improvement of Mathematical Critical Thinking

To examine improvements in students' mathematical critical thinking, the normalized gain (N-gain) was calculated based on the pretest and posttest scores. Fig. 7 illustrated the comparison of n-gain between experimental and control groups.

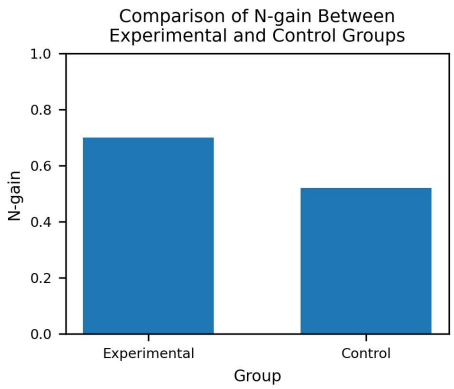


Fig. 7. Comparison of n-gain between experimental and control groups.

In addition to mean N-gain values, standard deviations and confidence intervals were reported to describe the variability and precision of improvement within each group. The descriptive statistics of N-gain are presented in Table 12.

Table 12. Descriptive statistics of N-Gain scores

Group	Mean N-gain	Standard deviation	95% Confidence Interval	Interpretation
Experimental	0.70	0.13	0.66–0.74	High
Control	0.52	0.15	0.47–0.57	Medium

The improvement in students' mathematical critical thinking was further analyzed using the normalized gain (N-gain). Based on the mean pretest and posttest scores, the experimental group obtained an N-gain value of 0.70, which falls into the high improvement category. In contrast, the control group achieved an N-gain value of 0.52, categorized as medium improvement.

These findings demonstrate that students who participated in the GeoGebra-supported Propose-STEM learning model experienced greater improvement in mathematical critical thinking than those who received conventional instruction. The integration of GeoGebra within the Propose-STEM learning model also fostered a learning environment that promoted conceptual exploration and higher-order mathematical reasoning.

D. Improvement of Mathematical Critical Thinking across Indicators

The improvement in students' mathematical critical thinking was further analyzed across four indicators: interpretation, analysis, evaluation, and inference. The findings show that all indicators improved following the implementation of the GeoGebra-supported Propose-STEM learning model. Fig. 8 presents the comparison of pretest and posttest scores across the four indicators.

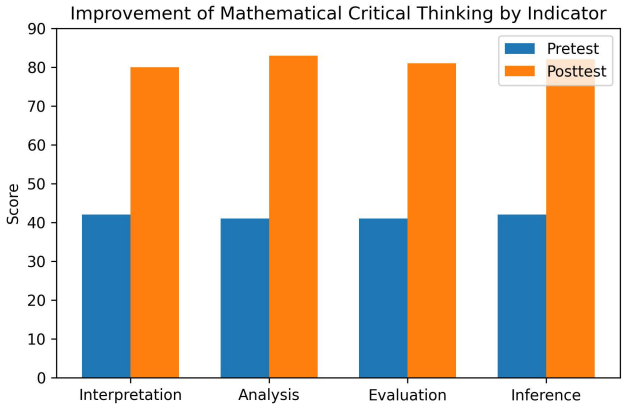


Fig. 8. Improvement of students' mathematical critical thinking by indicator.

Among the four indicators, analysis and inference demonstrated the greatest improvement, with N-gain values classified as high. This finding suggests that GeoGebra-supported learning activities particularly facilitated students' ability to analyze mathematical relationships and formulate logical conclusions based on graphical and symbolic representations.

Meanwhile, the interpretation and evaluation indicators showed moderate improvement. These results suggest that students became more capable of interpreting mathematical information and evaluating solution strategies after participating in the technology-supported learning environment.

Overall, the improvement across all four indicators demonstrates that the GeoGebra-supported Propose-STEM learning model supported the development of multiple dimensions of mathematical critical thinking, particularly in analytical and inferential reasoning.

E. Independent Sample Test

As a supplementary analysis, an independent-samples t-test was conducted to examine differences in posttest mathematical critical thinking scores between the experimental and control groups. However, because baseline equivalence was not fully achieved, interpretation of group differences primarily relied on ANCOVA-adjusted results.

The normality of the data distribution was tested using the Shapiro–Wilk test. As shown in Table 13, all datasets were normally distributed ($p > 0.05$).

Table 13. Shapiro–Wilk normality test results

Group	W		Sig. (p)	
	Pretest	Posttest	Pretest	Posttest
Control	0.952	0.958	0.121	0.195
Experimental	0.941	0.943	0.051	0.056

Table 14 presents the results of Levene's test for homogeneity of variances. The findings showed that the variance across groups was homogeneous for both the pretest and posttest data ($p > 0.05$). Although the assumptions for independent-samples t-test were satisfied, the results should be interpreted with caution because the groups differed significantly at baseline.

Table 14. Results of Levene's test for homogeneity of variances

Test	F	Sig. (p)
Pretest	1.844	0.179
Posttest	3.377	0.070

After the assumptions of normality and homogeneity were satisfied, an independent samples t-test was performed on the posttest scores. The results are presented in Table 15.

Table 15. Independent sample t-test

Variable	N	df	t	Sig. (p)
Experimental (mathematical critical thinking)	37	71	6.29	<0.001
Control (conventional Instruction)	36			

The analysis revealed a statistically significant difference between the experimental and control groups ($t = 6.29$, $p < 0.001$). This finding indicates that the experimental group demonstrated higher observed posttest performance in mathematical critical thinking than the control group. However, because the groups were not equivalent at baseline, these results should be interpreted together with the ANCOVA-adjusted findings, which provide a more appropriate estimate of the instructional effect. Consistent with the ANCOVA results, participation in the GeoGebra-supported Propose–STEM learning model was associated with higher mathematical critical thinking outcomes than conventional instruction.

F. Effect Size Analysis

To further examine the magnitude of the difference between groups, effect size analysis was conducted to complement statistical significance testing. Because the experimental and control groups were not fully equivalent at baseline, effect size interpretation was considered together with the ANCOVA-adjusted results.

Table 16. Effect size

Comparison	Mean difference	Cohen's d	Interpretation
Experimental (mathematical critical thinking)	13.68	1.48	Large
Control (Conventional Instruction)			

Based on the raw posttest mean difference and pooled standard deviation, the calculated effect size was Cohen's d = 1.48, as presented in Table 16. Although this value indicates a large standardized mean difference, it should be interpreted cautiously because it does not account for differences in pretest performance between groups.

According to established benchmarks [78], this value represents a large effect size and indicates a substantial difference in observed learning outcomes between groups. Consistent with the ANCOVA results, the large effect size suggests that students participating in the GeoGebra-supported Propose-STEM learning model demonstrated higher outcomes in mathematical critical thinking than students receiving conventional instruction. However, because the estimate was derived from raw posttest comparisons, the ANCOVA-adjusted effect size (partial $\eta^2 = 0.270$) was considered more appropriate for interpreting the magnitude of the instructional effect.

V. Discussion

The findings of this study provide evidence that integrating the GeoGebra-supported Propose-STEM learning model was associated with greater improvements in students' mathematical critical thinking than conventional instruction. The higher normalized gain and large effect size indicate that the proposed instructional approach was associated with improved learning outcomes and meaningful practical implications, supporting the development of higher-order thinking skills. These findings reinforce the argument that the effectiveness of technology-enhanced learning depends not merely on the use of digital tools but on their systematic integration within well-structured pedagogical frameworks.

In addition to these findings, ANCOVA further strengthens the study's internal validity by statistically controlling baseline differences between groups. Although the pretest results indicated that the groups were not equivalent at baseline, the ANCOVA analysis showed that differences in posttest performance remained statistically significant after controlling for initial performance.

Because baseline equivalence was not achieved, interpretation of intervention magnitude was based primarily on ANCOVA-adjusted estimates rather than raw posttest comparisons. Although the observed Cohen's d value indicated a large standardized mean difference, this estimate should be interpreted with caution because it does not account for initial group differences. Therefore, the partial eta squared obtained from ANCOVA was considered a more appropriate indicator of instructional effect size in the present study.

The adjusted effect size suggested that the GeoGebra-supported Propose-STEM learning model continued to make a meaningful contribution to students' mathematical critical thinking after controlling for baseline performance. In addition, the inclusion of confidence intervals and variability measures for N-gain provided a more comprehensive interpretation of learning improvement across groups. Taken together, these findings suggest that the observed differences were not solely explained by pre-existing differences and were associated with participation in the GeoGebra-supported Propose-STEM learning model.

One key factor underlying this improvement is GeoGebra, an interactive mathematical environment that enables students to explore mathematical relationships through multiple dynamic representations. Unlike traditional

instructional approaches that rely on static representations, GeoGebra allows students to examine algebraic, graphical, and numerical forms simultaneously, thereby supporting deeper conceptual understanding. This finding is consistent with previous studies indicating that dynamic mathematics environments enhance students' visualization and analytical reasoning abilities [30, 66, 79]. However, this study extends prior research by demonstrating that such technological affordances, when systematically integrated within a structured pedagogical framework, may support the development of mathematical critical thinking rather than merely improving conceptual understanding.

From a cognitive perspective, GeoGebra's effectiveness can be explained by its ability to support iterative cycles of conjecture, experimentation, and verification. When students manipulate mathematical objects and receive immediate visual feedback, they can evaluate their reasoning in real time, which promotes deeper engagement with mathematical structures. This aligns with research in technology-enhanced learning, which suggests that interactive environments facilitate the development of higher-order thinking by enabling students to actively construct and test mathematical ideas [80, 81]. In the present study, this process appears to contribute particularly to improvements in the analysis and inference indicators, where students are required to examine relationships among variables and draw logical conclusions.

In addition to technological support, integrating problem-posing activities within the Propose-STEM framework may enhance students' critical thinking. Unlike conventional problem-solving approaches, problem posing requires students to generate, modify, and extend mathematical problems, thereby engaging them in deeper analysis of mathematical structures. Previous studies have reported that problem posing promotes creativity and reasoning [24, 82]. The findings of this study support and extend this perspective by showing that combining problem-posing with technology-supported exploration can further enhance students' ability to evaluate and construct mathematical arguments.

Furthermore, the integration of STEM-oriented learning provides an interdisciplinary context that strengthens the relevance of mathematical reasoning. By engaging students in contextual engineering problems, the learning process encourages them to apply mathematical concepts in authentic situations, thereby enhancing their ability to interpret quantitative information and evaluate solution strategies. This finding is consistent with previous research indicating that STEM-based learning environments promote critical thinking through inquiry and problem solving [7, 40]. However, this study contributes to the literature by demonstrating that the effectiveness of STEM learning may be further supported when combined with structured problem posing and dynamic visualization tools.

The effectiveness of the Propose-STEM learning model stems from the integration of multiple instructional components. While previous studies have examined PBL, problem posing, STEM, and technology integration separately, the present study does not directly compare these approaches in isolation. Therefore, the potential advantages of combining these elements should be interpreted with caution, taking into account the existing literature. Nevertheless, the findings suggest that the integrated approach provides a meaningful learning environment for developing mathematical critical thinking when compared to conventional instruction.

From a pedagogical perspective, these findings suggest that mathematics instruction in engineering education should move beyond procedural teaching toward technology-supported, problem-based learning environments. The integration of dynamic visualization tools, problem posing, and interdisciplinary contexts provides students with opportunities to engage more deeply with mathematical concepts and to develop critical thinking skills essential to solving complex engineering problems.

Overall, the findings suggest that the GeoGebra-supported Propose-STEM learning model provides a promising instructional approach for supporting mathematical critical thinking in engineering mathematics education. By integrating dynamic technology, problem-based learning, and problem-posing activities, the model may support students' conceptual understanding and foster the development of analytical reasoning skills required in real-world engineering contexts.

A. Limitations of the Study

Despite the promising findings, several limitations of this study should be acknowledged. First, the study employed a quasi-experimental design using intact classes, which did not allow for random assignment of participants. Consequently, baseline equivalence between groups was not fully achieved, and pre-existing differences may have influenced the observed outcomes. Although Analysis of Covariance (ANCOVA) was employed to statistically adjust for differences in pretest performance and estimate adjusted posttest outcomes, causal interpretation should still be made with caution. In addition, the Hawthorne effect cannot be entirely ruled out, as students in the experimental group were aware that they were participating in an instructional intervention involving a new learning model supported by GeoGebra.

Second, the intervention was conducted by the same instructor for both groups to maintain instructional consistency and reduce potential instructor-related variability. However, minor variations in instructional delivery, classroom interaction, and student engagement may still have influenced the learning outcomes. Although implementation fidelity was supported by structured lesson plans, minor differences in classroom dynamics may have occurred during the intervention.

Third, the study relied on a single measurement instrument to assess mathematical critical thinking. Although the instrument demonstrated high validity and reliability, future studies may benefit from incorporating multiple assessment methods that could provide a more comprehensive evaluation of students' abilities.

Fourth, the findings of this study are limited to short-term effects, as the posttest was administered immediately after the intervention. Therefore, the long-term impact of the GeoGebra-supported Propose-STEM learning model on students' mathematical critical thinking remains unclear.

Finally, the study was conducted within a single institutional context and focused on calculus topics, which

may limit the generalizability of the findings to other educational settings or mathematical domains.

VI. Conclusion

This study investigated the effectiveness of the GeoGebra-supported Propose-STEM learning model in enhancing students' mathematical critical thinking in engineering mathematics courses. The findings suggest that implementing this model was associated with higher students' mathematical critical thinking performance than conventional instruction, as evidenced by higher posttest scores, a larger normalized gain, and a large effect size. These results further suggest that integrating dynamic visualization tools with inquiry-based and problem-posing learning strategies may support the development of higher-order mathematical reasoning.

The findings further indicate that integrating GeoGebra-supported visualization, problem-based learning, problem-posing activities, and STEM-oriented learning provides an instructional framework that supports students' abilities to interpret, analyze, evaluate, and construct mathematical arguments. This study contributes to the literature by demonstrating that integrating these approaches within a coherent instructional framework provides more meaningful support for the development of mathematical critical thinking than conventional instruction. From a practical perspective, the results suggest that technology-supported, student-centered learning environments may offer valuable opportunities to foster critical thinking skills in engineering education.

Overall, this study confirms that the GeoGebra-supported Propose-STEM learning model offers a promising, innovative approach to enhancing mathematical critical thinking and makes a meaningful contribution to advancing technology-enhanced mathematics education in engineering contexts.

Appendix

Example of Mathematical Critical Thinking Assessment Item, Sample Student Response, and Rubric Application.

To improve transparency of the assessment procedure, this appendix presents an example of the mathematical critical thinking assessment instrument, an anonymized student response, and an illustration of rubric application.

A. Sample Assessment Item

In an urban drainage design project, the flow velocity distribution along an open trapezoidal channel is modeled by the following function:

$$V(x) = -0.004x^3 + 0.09x^2 + 0.6x + 1.2$$

where:

$V(x)$ = flow velocity (m/s)

x = distance from the channel inlet (m), with $0 \le x \le 20$

To satisfy hydraulic engineering requirements, the channel must maintain a safe and stable flow condition throughout its length. Specifically:

- Flow velocity must remain below 3.0 m/s to prevent scouring of the channel bed
- Flow velocity must remain above 0.6 m/s to prevent sedimentation
- The rate of velocity change $|V'(x)|$ must not exceed 0.5 m/s per meter at any point

Tasks:

(1) Interpret the meaning of the function $V(x)$ and identify all mathematically and hydraulically relevant information, including initial conditions, domain constraints, and engineering parameters.

(2) Determine the location(s) along the channel where the flow velocity reaches local maximum or minimum values. Show the complete differentiation procedure and justify your classification of each critical point.

(3) Evaluate whether the resulting velocity profile satisfies all three engineering requirements listed above. Support your evaluation with mathematical evidence at all critical points and at the boundaries of the domain.

(4) Based on your analysis, provide a recommendation for improving the channel design. Propose a modified velocity function or an adjustment to the channel geometry and justify your recommendation mathematically.

Targeted Indicators: Interpretation, Analysis, Evaluation, Inference

B. Sample Student Response

1) Indicator 1. Interpretation

The function $V(x) = -0.004x^3 + 0.09x^2 + 0.6x + 1.2$ models the flow velocity along a drainage channel over the domain $0 \le x \le 20$ m.

Relevant mathematical information:

- At $x = 0$: $V(0) = 1.2$ m/s — this is the initial flow velocity at the channel inlet
- The function is a cubic polynomial, meaning the velocity will rise, reach a peak, and potentially decline over the domain
- The domain is bounded: [0, 20] m, so both endpoints must be evaluated alongside any interior critical points
- Three engineering constraints are given: $V < 3.0$ m/s (anti-scour), $V > 0.6$ m/s (anti-sedimentation), and $|V'(x)| \le 0.5$ m/s per meter (rate of change limit)

Engineering assumptions: The model assumes steady-state flow conditions and a uniform channel cross-section along x. These assumptions are necessary to justify representing velocity as a smooth, continuous function of position alone.

2) Indicator 2. Analysis

The function $V(x) = -0.004x^3 + 0.09x^2 + 0.6x + 1.2$ models the flow velocity along a drainage channel over the domain $0 \le x \le 20$ m.

First derivative:

$$V'(x) = -0.012x^2 + 0.18x + 0.6$$

Setting $V'(x) = 0$ and simplifying:

$$x^2 - 15x - 50 = 0$$

Applying the quadratic formula:

$x = (15 \pm \sqrt{425}) / 2 \Rightarrow x_1 \approx 17.81$ m, $x_2 \approx -2.81$ m (outside domain — rejected)

Only $x \approx 17.81$ m lies within [0, 20]. Applying the second derivative test:

$$V''(x) = -0.024x + 0.18 \implies V''(17.81) = -0.247 < 0$$

∴ Local Maximum

Velocity at critical point:

V (17.81) ≈ 17.82 m/s

3) Indicator 3. Evaluation

Velocity at domain boundaries:

V (0) = 1.2 m/s

V (20) = –32 + 36 + 12 + 1.2 = 17.2 m/s.

4) Indicator 4. Inference

The fundamental problem is that the cubic and quadratic coefficients together produce an excessively steep velocity rise over the 20 m domain. Two adjustments are recommended:

Recommendation 1: Reduce the initial velocity gradient by modifying the linear coefficient from 0.6 to 0.3, so that $V'(0) = 0.3 < 0.5$.

Recommendation 2: Reduce peak velocity by scaling down the quadratic term. A candidate modified function is:

$$V^*(x) = -0.004x^3 + 0.045x^2 + 0.3x + 1.2$$

Verification: $V^{*\prime}(0) = 0.3 < 0.5$ ✓. Setting $V^{*\prime}(x) = 0$ yields $x = 10$ m, giving $V^*(10) = 4.7$ m/s. This still exceeds 3.0 m/s, indicating that a more conservative coefficient set or channel segmentation is needed. The student recommends re-deriving the velocity model from measured discharge data rather than using a generic polynomial fit.

C. Rubric Application

The assessment rubric used to evaluate the sample student response is presented in Table A1.

Table A1. Sample Scoring Rubric for Assessing Mathematical Critical Thinking

Indicator	Criteria	Score
Interpretation	Student identifies all relevant mathematical information (initial value, domain, critical constraints) and states hydraulic assumptions with clear engineering justification.	4 / 4
Analysis	Student correctly differentiates $V(x)$, applies the quadratic formula, rejects the out-of-domain root with justification, and classifies the critical point using the second derivative test.	4 / 4
Evaluation	Student systematically checks all three engineering requirements at both critical points and domain boundaries, presents results in a structured table, and identifies two specific violations.	4 / 4
Inference	Student correctly identifies the source of the problem and proposes a modified function with partial verification, but the modified function still violates the anti-scour requirement without a fully resolved alternative.	3 / 4
Total		15 / 16

The student earned 15 out of 16 points. Full marks were awarded for Interpretation, Analysis, and Evaluation: the student demonstrated thorough identification of all relevant variables and constraints, executed a complete and correct differentiation procedure including proper rejection of the extraneous root, and conducted a systematic multi-point evaluation against all three engineering thresholds. One point was deducted on Inference because while the student correctly diagnosed the root cause and proposed a mathematically grounded modification, the proposed function $V^*(x)$ was shown to still violate the scouring constraint, and no fully valid alternative was resolved within the response. A complete Inference response would either derive coefficients that provably satisfy all three requirements or formally bound the parameter space within which all constraints are simultaneously met.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Windry Novalia Jufri contributed to conceptualization, methodology, investigation, data curation, formal analysis, visualization, and the preparation of the original draft. I Made Arnawa was responsible for supervision, conceptualization, methodology, validation, and manuscript review and editing. Edwin Musdi contributed to methodology, formal analysis, validation, and manuscript review and editing. Rijal Abdullah contributed to investigation, data curation, and resources. Abna Hidayati was involved in validation, manuscript review and editing, and visualization. Ari Syaiful Rahman Arifin contributed to data curation, investigation, and resources. All authors reviewed and approved the final version of the manuscript.

Acknowledgment

The authors would like to express their sincere gratitude to the Department of Civil Engineering and the Graduate School of Universitas Negeri Padang for their support and facilitation during the completion of this research. The authors also thank the students who participated in this study for their active involvement and valuable contributions during the implementation of the learning activities. Their participation greatly supported the successful completion of this research.

The authors are also grateful to colleagues and academic staff who provided valuable suggestions and constructive feedback that helped improve the quality of this study and the preparation of this manuscript.

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