Reconstructing GeoGebra-based Geometry Learning within the Praxeology from the Anthropological Theory of the Didactic to Enhance Students' Problem-solving Skills

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***Abstract*—This study aims to reconstruct GeoGebra-based geometry learning within praxeology from Anthropological Theory of the Didactic (ATD) to enhance students' problem-solving skills. It seeks to redesign the learning process by integrating digital tools and didactical praxeologies to strengthen students' conceptual and reflective understanding in geometry. The research design used is a quasi-experimental Non-Equivalent Control Group Design with two classes, namely the experimental class (ATD-GeoGebra learning) and the control class (conventional ATD learning). The research subjects consisted of 60 mathematics education students who were selected purposively. The research instrument was a descriptive test of geometry problem-solving skills covering five indicators: problem identification, strategy planning, technique implementation, result interpretation, and evaluation-reflection. The results showed that the integration of GeoGebra significantly improved students' problem-solving skills compared to ATD learning without digital support. The average N-Gain score in the experimental class reached 69.02% (high category), while in the control class it was only 42.96% (medium category). Independent t-tests on all indicators showed significant differences with a value of *p* < 0.001. These results confirm that the integration of GeoGebra strengthens the connection between the praxeological aspects (task–technique–technology–theory) in ATD, thereby transforming geometry learning from procedural to reflective and conceptual. Thus, the ATD-GeoGebra model is recommended as an effective innovative learning design to improve students' problem-solving skills and mathematical literacy in higher education.**

***Keywords*—GeoGebra, anthropological theory of didactics, geometry, problem solving, digital learning**

I. Introduction

The paradigm shift in mathematics education in the digital age requires a fundamental transformation in the way we think, teach, and solve mathematical problems. The urgency of this research lies in the increasing need for learning approaches that not only emphasize concept transfer but also foster students' reflective, creative, and analytical thinking skills. In the context of geometry, the integration of digital technology such as GeoGebra plays a significant role in transforming conventional learning into visual and constructive exploration-based learning [1, 2]. This approach is in line with the Anthropological Theory of the Didactic (ATD), which places the learning process as a systematic relationship between task, technique, technology, and theory [3]. The increasing complexity of geometry problems at the higher education level demands technology-based didactic innovations to improve learning effectiveness [4–6]. Thus, this research is relevant to address the need for contextual, interactive, and problem-solving-oriented geometry learning.

The main problem in learning geometry in higher education is still related to students' low ability to connect conceptual representations with visual representations. Based on previous research reports, students often have difficulty understanding the relationships between geometric concepts, especially when asked to explain the logical reasons behind visual procedures [7–9]. These obstacles are exacerbated by teaching practices that are still lecturer-centered and lack technological exploration [10]. Several studies also show that the integration of GeoGebra has not been optimally utilized in a deep theoretical context, especially in the ATD framework which emphasizes praxeological analysis as the basis for learning [11]. In the context of university-level geometry, where students are required not only to manipulate geometric objects but also to justify techniques and interpret results, the praxeological structure of tasks, techniques, technologies, and theories becomes essential. Integrating praxeology from ATD into geometry instruction provides a coherent analytical lens to align GeoGebra-based activities with explicit mathematical tasks and underlying justifications, thereby reducing the gap between conceptual understanding and problem-solving performance. For this reason, the integration of praxeology from ATD is not merely complementary but necessary to ensure that GeoGebra functions as a didactic tool rather than a purely exploratory visualization aid.

Initial observations conducted on fourth-semester prospective mathematics teachers at a private university in West Nusa Tenggara Province, Indonesia, using a problem-solving test adapted from the test used in this study, showed that most students had difficulty identifying steps to solve geometry problems based on dynamic visualizations. The observation data are summarized in Table 1.

The results in Table 1 show that students' problem-solving abilities were in the low category, especially in the indicators of planning and evaluating solutions. This phenomenon reveals the existence of learning obstacles [12, 13] and epistemological gaps in the application of geometric concepts that require high spatial representation. Therefore, a learning approach that stimulates reflective and interactive visual

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activities simultaneously is needed so that students are able to independently develop concepts and problem-solving.

Table 1. Initial observation results of students' geometry problem-solving skills ($n = 40$)

Problem-Solving Ability Indicators	Percentage of Students (%)	Category
Understanding the problem	62.5	Moderate
Planning a solution strategy	45.0	Low
Implementing the strategy	40.0	Low
Interpretation of Results	42.6	Low
Evaluation and reflection	35.0	Low
Overall average	45.02	Low

The solution proposed in this study is the application of praxeology from the Anthropological Theory of the Didactic (ATD) integrated with GeoGebra as an exploratory tool. This theoretical choice is grounded in the distinctive strength of praxeology in analyzing learning as a structured system of tasks, techniques, underlying technologies, and theoretical justifications. Unlike cognitive or constructivist learning theories that primarily emphasize individual mental processes or general instructional strategies, praxeology from ATD enables a fine-grained examination of how mathematical knowledge is produced, justified, and practiced within specific didactic tasks. This makes it particularly suitable for investigating GeoGebra-based geometry learning, where students' problem-solving activities are closely tied to the use of digital techniques and their mathematical rationales. Therefore, praxeology from ATD provides an analytical lens that other theories do not sufficiently offer, especially in capturing the interplay between tasks, digital techniques, and conceptual understanding in geometry learning.

This approach is designed to strengthen the relationship between mathematical activities (praxeology) and concrete digital representations. In ATD, learning is understood as a system of relationships between tasks (mathematical tasks), techniques (completion strategies), technology (conceptual or digital tools), and theory (basic scientific principles) [3]. The integration of GeoGebra in these four dimensions is expected to optimize students' visualization, analysis, and generalization abilities [14–16]. Thus, this model serves as a means to transform learning from mere procedural application into an exploratory experience that links theory with problem-solving practice.

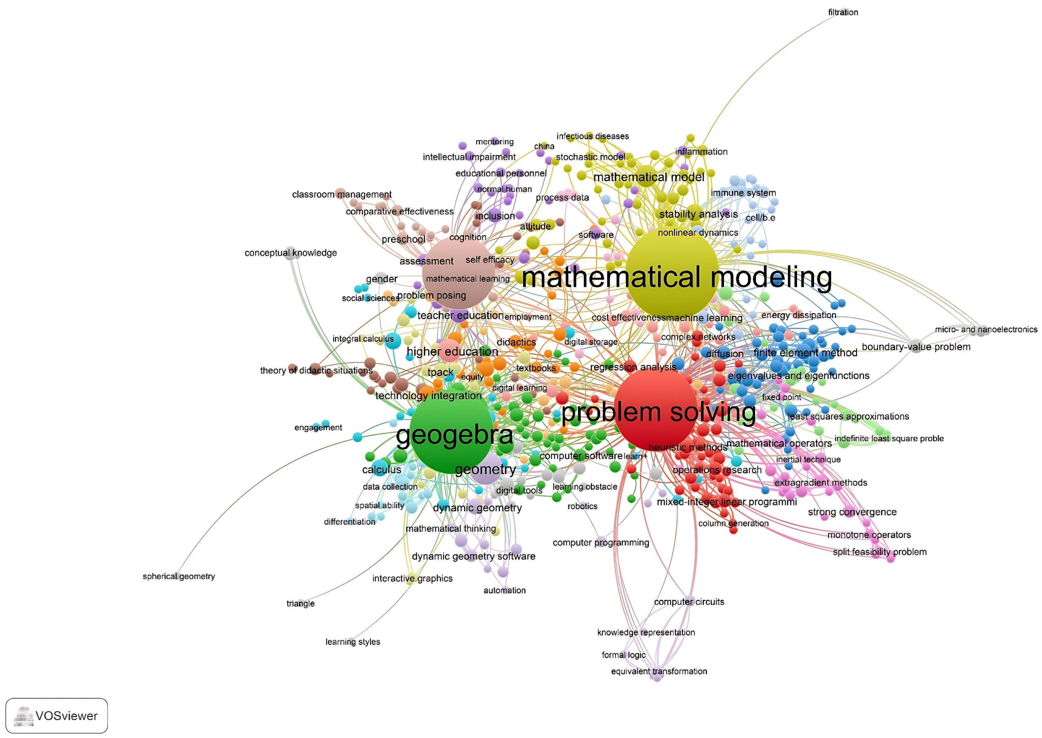

Fig. 1. Bibliometric map of GeoGebra and ATD research based on 6225 scopus documents (VOSviewer, 2025).

Bibliometrically, Fig. 1 above shows that an analysis of 6,225 Scopus documents indicates that research on GeoGebra and geometry learning has grown rapidly, but its main focus is still on the effectiveness of visual media, strengthening spatial concepts, and increasing learning motivation. The network map visualization analyzed through VOSviewer shows five main clusters: GeoGebra and digital mathematics education, didactic theory, problem solving, teacher training, and mathematical modeling. The relationship between the Anthropological Theory of the Didactic (ATD) and the use of GeoGebra is still relatively weak, as indicated by the low connectivity between nodes in the theoretical cluster [17, 18]. This indicates a significant research gap, namely the lack of research examining the theoretical integration between the anthropological didactic approach and the use of dynamic geometry-based technology. This gap is not merely empirical but fundamentally theoretical. The integration of praxeology from the Anthropological Theory of the Didactic (ATD) and dynamic geometry technology is required not only because previous studies rarely combine them, but because each addresses complementary limitations in higher-level geometry learning. Praxeology provides an analytical framework to examine mathematical tasks, techniques, technologies, and underlying justifications, while dynamic geometry environments such as GeoGebra operationalize these components through interactive visualization and experimentation. When applied independently, praxeological analysis risks remaining abstract, and technology-based instruction risks becoming procedurally driven. Their integration therefore becomes necessary to ensure conceptual coherence, epistemic transparency, and meaningful problem-solving development in advanced mathematics learning. Therefore, this study occupies an important position in strengthening the relationship between didactic theory, digital technology, and the effectiveness of higher-level mathematical learning.

The novelty of this study lies in its systematic attempt to integrate the praxeology from the ATD with the GeoGebra digital tool to develop a geometry learning system oriented toward improving problem-solving skills. Unlike previous studies that tended to separate theoretical and technological aspects, this study empirically tests the effectiveness of their interaction in the context of higher education [19–21]. A bibliometric analysis of 6225 Scopus publications shows that the relationship between didactic theory and GeoGebra integration is still at a conceptual stage and has not been extensively researched in the quasi-experimental realm that measures higher-order cognitive achievements. Therefore, this study contributes to the development of a technology-embedded didactics model that bridges theory and practice in dynamic visualization-based geometry learning. This study not only expands the scope of ATD into the digital domain, but also builds a theoretical foundation for evidence-based mathematics education.

Based on the background, problems, and research gaps that have been described, the research questions in this study are: (1) Is there a significant effect of integrating GeoGebra into geometry learning based on the Anthropological Theory of the Didactic (ATD) on students' problem-solving abilities? (2) To what extent is this learning model effective in improving mathematical problem-solving indicators? The objectives of this study are to: (1) test the effectiveness of GeoGebra integration in the context of anthropological didactics theory on improving geometry problem-solving abilities, and (2) provide empirical contributions to the development of learning models based on didactics theory and digital technology in higher education.

II. Literature Review

A. GeoGebra Application

The integration of GeoGebra in geometry learning has become a major focus in the development of modern mathematics pedagogy due to its ability to visualize abstract concepts and strengthen students' cognitive construction. GeoGebra functions as a dynamic geometry environment that allows simultaneous interaction between algebraic, graphical, and numerical representations, thereby facilitating the conceptual transition from visualization to mathematical abstraction [2, 10, 22]. Research by Roanes-Lozano and Solano-Macías [18] shows that the use of GeoGebra can increase learning engagement through an exploratory approach, especially in three-dimensional geometry concepts. Furthermore, Kartal and Çinar [14] prove that the integration of GeoGebra contributes significantly to the development of Technological Pedagogical Content Knowledge (TPACK) in prospective mathematics teachers, showing that technology not only functions as a visual aid but also as an epistemological means of understanding mathematical conceptual structures. In the context of geometry learning, the GeoGebra application strengthens problem-solving skills by fostering spatial visualization and deductive reasoning [7, 15, 16]. Similar research by Restrepo-Ochoa *et al.* [23] confirms that GeoGebra supports van Hiele's model in developing proportionality and geometric representation. Thus, theoretically and empirically, GeoGebra has been proven to strengthen the bridge between constructivist cognitive theory and exploration and discovery-based learning practices.

However, despite numerous studies highlighting the effectiveness of GeoGebra in improving learning outcomes, there are still conceptual gaps in its integration with more philosophical didactic theories such as the Anthropological Theory of the Didactic (ATD). Studies by Artigue [1] and Trouche as well as Yunianta *et al*. [24] emphasize that the development of didactic design requires a coherent theoretical framework grounded in the Anthropological Theory of the Didactic, in which praxeology comprising tasks, techniques, technologies, and theories serves as the core analytical unit for examining mathematical activity and learning processes. A meta-analysis study by Suparman *et al*. [25] shows that most studies still focus on the cognitive effects of GeoGebra without deeply examining the praxeological process in the formation of mathematical knowledge. In addition, Prince [26] and Aliu *et al.* [27] confirm that the integration of GeoGebra has a positive effect on student motivation and confidence, but there has not been much research exploring its epistemic and didactic implications in a university context. Therefore, this study attempts to fill this gap by combining GeoGebra as an interactive digital medium in ATD-based geometry learning to encourage problem-solving skills that are not only procedural but also reflective and conceptual, while expanding the theoretical basis of the relationship between digital technology and contemporary didactic theory in mathematics education.

B. Anthropological Theory of the Didactic (ATD)

The Anthropological Theory of the Didactic (ATD) developed by Yves Chevallard is one of the most influential theoretical frameworks in the analysis of mathematical activities and the relationship between teachers, students, and knowledge. The praxeology from the Anthropological Theory of the Didactic (ATD) is particularly appropriate for geometry learning because geometry inherently involves the coordination of tasks, techniques, justifications, and theoretical reasoning. Geometric activities such as constructing figures, exploring properties, and validating conjectures require not only procedural execution but also explicit technological discourse that explains why a technique works and how it is grounded in geometric theory. Within this context, praxeology conceptualizes mathematical learning as an integrated system consisting of task (T), technique (τ), technology (θ), and theory (Θ), allowing geometric problem solving to be analyzed as a structured and explainable knowledge practice rather than as isolated skills. While other learning theories may emphasize cognitive processes or instructional strategies, praxeology from ATD is especially suitable for geometry because it captures the epistemic structure of geometric knowledge and the role of digital tools, such as GeoGebra, in mediating the transition from empirical exploration to theoretical justification [1]. This approach places the didactic process as an anthropological phenomenon in which every learning action is influenced by social, epistemological, and institutional contexts [28, 29]. Alves *et al.* [19] and Novotná and Hošpesová [3] shows that this theory provides a systematic framework for understanding how mathematical activities are designed and implemented effectively through the construction of didactic situations. In the context of geometry learning, ATD plays an important role in explaining how students construct concepts through the relationship between symbolic representations and technical practices. Castro *et al*. [30] and Hernández-Rodríguez *et al*. [31] emphasize that the application of ATD helps identify didactic transposition, which is the shift of knowledge from scientific form to teachable form, thereby strengthening the epistemological foundation in mathematics education. Thus, ATD is not only an analytical theory, but also functions as a design framework that enables the design of learning that is oriented towards problem solving, epistemic reflection, and collaboration.

Although the effectiveness of ATD has been widely studied, the relationship between this theory and the integration of digital technologies such as GeoGebra is still relatively new and has not been comprehensively explored. Daher *et al.* [32] and Yenil *et al.* [33] shows that integrating ATD principles with digital media can enrich the design of didactic situations through interactivity, visualization, and automatic feedback that deepens understanding of geometric concepts. However, a meta-synthesis conducted by Hortelano and Prudente [17] reveals that most research is still limited to the elementary and secondary school levels, while applications in the context of higher education have not yet produced stable empirical models. Tonra *et al.* [12] and Yunianta *et al.* [24] also confirm that ATD-based learning requires integration with digital tools to overcome complex learning obstacles in topics such as limits and spatial geometry. Against this backdrop, this study offers a novel contribution through the application of GeoGebra as an interactive digital medium in the context of ATD to strengthen the relationship between students' practical and cognitive dimensions. This relationship is explored through field data collection with problem-solving tests, observations, and pretest-posttest procedures. This approach is expected to not only improve problem-solving skills but also shift the paradigm of geometry learning into a scientific activity that encourages epistemological reflection, in line with the new direction of digital didactic theory and the transformation of mathematics learning in the 5.0 era.

C. *Students' Problem-Solving Skills*

Mathematical problem-solving skills are an essential indicator of higher-order thinking skills, which include the processes of understanding problems, designing strategies, implementing solutions, and reflecting on results [34]. In the context of higher education, this ability is the foundation for developing numeracy, logical reasoning, and mathematical creativity in prospective educators [35, 36]. Abdüsselam *et al.* [37] confirms that problem solving not only requires algorithmic skills, but also involves epistemic and metacognitive aspects that foster awareness of one's own thinking processes. This perspective is reinforced by Hernández-Rodríguez *et al*. [38], who found that students with high problem-solving abilities demonstrate reflective capacity in connecting didactic situations with applied contexts. In geometry learning, this ability requires students to understand the relationship between visual and conceptual representations [23], while Castro-Rodríguez *et al.* [11] highlight the role of didactic modeling in facilitating the transition from procedural to conceptual understanding. Thus, students' problem-solving abilities are not only a function of intensive practice, but also the result of a complex integration of learning experiences, epistemological contexts, and technological support that mediates cognitive exploration.

A number of recent studies show that students' problem-solving abilities can be improved through the integration of constructivist-oriented didactic approaches and digital media. Sari *et al*. [39] and Senjayawati *et al.* [40] found that didactic designs based on probabilistic theory and lesson study can minimize learning barriers while optimizing students' conceptual reflection. Fardian *et al.* [41] confirms that this ability is greatly influenced by the epistemic structure of teaching materials that are systematic and praxeology-based, while Aiyub *et al.* [42] add that the hermeneutic process in problem solving requires a phenomenological understanding of non-routine mathematical thinking patterns. Furthermore, Torkildsen *et al.* [43] and Geiger *et al.* [44] reveal that international research trends emphasize the need for a cross-theoretical approach that combines cognitive, affective, and didactic models in explaining problem-solving performance. Therefore, this study places students' problem-solving abilities as a central variable developed through the integration of the Anthropological Theory of the Didactic (ATD) and GeoGebra. This approach is expected to produce a learning model that is not only cognitively effective but also reflective and transformative, expanding the problem-solving paradigm from mere algorithmic reproduction to the construction of meaningful and contextual mathematical knowledge for future educators.

III. Materials and Methods

A. *Research Types and Designs*

This study uses a quantitative approach with a quasi-experimental design of the Non-Equivalent Control Group Design type. This design was chosen based on the consideration that the class conditions in the research population had already been established, so that the researchers could not randomly assign participants. However, this design allows for empirical testing of differences in learning outcomes between the experimental and control groups through different treatments, with strict control of external variables that may affect the research results [45, 46]. The experimental group used ATD with GeoGebra integration, while the control group used ATD with textbook integration as a source of practice questions.

A quasi-experimental approach is considered relevant to test the effectiveness of integrating GeoGebra into geometry learning based on the Anthropological Theory of the Didactic (ATD) on students' problem-solving abilities. The experimental class received an intervention in the form of ATD learning integrated with GeoGebra (X_1), while the control class received ATD learning without GeoGebra integration (X_2). Through this design, researchers can objectively assess the influence of GeoGebra technology on improving students' mathematical problem-solving abilities by considering praxeological, cognitive, and epistemological factors within the ATD framework. This can be seen in Table 2 below:

Table 2. Quasi-experimental research design of the non-equivalent control group type

Group	Pretest	Treatment	Posttest
Experiment	O_1	X_1	O_2
Control	O_3	X_2	O_4

where, X_1 = Geometry learning based on Anthropological Theory of the Didactic (ATD) with GeoGebra integration; X_2 = Geometry learning based on Anthropological Theory of the Didactic (ATD) without GeoGebra integration; O_1, O_3 = Pretest of problem-solving skills; and O_2, O_4 = Posttest of problem-solving skills.

B. Research Procedures

This research procedure was carried out in four main stages: (1) Planning, (2) Implementation, (3) Evaluation, and (4) Reflection on the research results.

In the planning stage, a preliminary study was carried out to identify students' initial problem-solving difficulties in geometry and to examine existing instructional practices. This preliminary study involved classroom observations, analysis of students' prior problem-solving test results, and discussions with lecturers. The findings revealed limited opportunities for exploratory reasoning, weak connections between geometric representations, and students' difficulties in articulating problem-solving strategies.

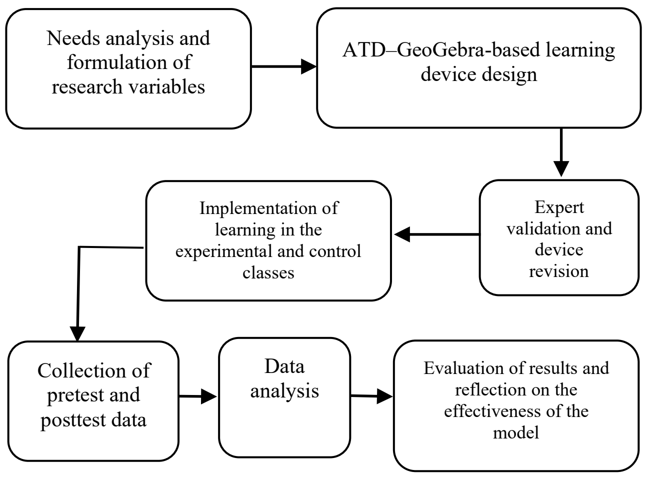

Fig. 2. Research procedure flowchart.

Based on these findings, several revisions were made to the learning design, including (a) restructuring learning tasks to align with the praxeology from ATD (tasks, techniques, and technologies), (b) integrating GeoGebra as an exploratory tool to strengthen the technological component of praxeology, and (c) refining problem-solving indicators used in the assessment instruments. These revisions informed the instructional design implemented in the experimental group during the implementation stage.

The planning stage included the development of ATD-based learning tools integrated with GeoGebra, expert validation, and testing of problem-solving ability instruments. The implementation stage was carried out in eight meetings in both classes with different treatments. The evaluation stage includes administering pre-tests and post-tests to measure the effectiveness of the intervention and analyzing learning outcomes using statistical software and the Rasch model. The reflection stage is carried out to assess the implementation of the ATD syntax, student responses, and the effectiveness of the instruments in achieving learning objectives. A summary of the intended research procedures is shown in Fig. 2.

C. Research Subjects or Participants

The research subjects consisted of two classes of mathematics education students selected using purposive sampling based on their initial abilities and availability of digital devices (simple reconstruction is presented in Table 3 below). The experimental class consisted of 30 students who participated in ATD-based learning with GeoGebra, while the control class consisted of 30 students who participated in conventional learning.

Table 3. Classification of research participants

Class	Number of Participants	Type of Treatment
Experiment	30	ATD + GeoGebra
Control	30	ATD

The inclusion criteria included active fourth-semester students who had taken basic geometry courses and had access to computers, while the exclusion criteria included students who did not attend at least 80% of the meetings or had extreme pretest scores.

D. Data Collection Techniques and Procedures

The data collection technique used two main instruments: a problem-solving ability test and a learning implementation observation sheet. The test was conducted twice a pretest and a posttest to measure the improvement in students' ability to identify, design strategies, and evaluate solutions to geometric problems. The pre-test and post-test were designed to be equivalent in terms of content, cognitive demand, and difficulty level. Both tests measured the same indicators of problem-solving performance, including problem comprehension, strategy formulation, procedural execution, and result interpretation. Although the items were presented with different numerical values and contextual representations, they were constructed based on the same test blueprint to ensure equivalence and to minimize memorization effects.

The data collection procedure was carried out in structured stages: (1) researchers administered a pretest before treatment to measure initial abilities; (2) ATD-GeoGebra-based learning was applied in the experimental class and conventional methods in the control class; (3) a posttest was administered after the learning cycle was completed; and (4) results were analyzed using inferential statistical tests to assess the effectiveness of the treatment. Data validation was strengthened through triangulation between sources (test results, observations, and field notes) to ensure consistency of results.

E. Data Collection Instruments

The research instrument consisted of an essay-based geometric problem-solving test designed to capture students' praxeological activities, namely task types, techniques, and underlying reasoning (presented in Table 4 below). The construction of the instrument was grounded in Polya's four problem-solving indicators understanding the problem, devising a plan, carrying out the plan, and looking back which were operationalized within the praxeology from the Anthropological Theory of the Didactic (ATD). This framework was adopted to ensure that each test item not only assessed procedural outcomes but also reflected students' techniques and justifications when engaging with geometric tasks. Sample test items are provided to illustrate how students were required to interpret geometric situations, select appropriate techniques (including GeoGebra-based constructions for the experimental group), and articulate their reasoning. Each indicator reflects higher-order thinking skills, namely: identifying problems, formulating strategies, applying problem-solving techniques, interpreting results, and evaluating solutions. The instrument was validated by three experts (an educational evaluation expert, an ATD expert, and a GeoGebra expert) and tested for reliability using Cronbach's Alpha and the Rasch model.

Table 4. Specifications of the problem-solving ability instrument

Code	Indicator	Specific Description	Question Context (5 items per indicator)	Score Weight
I1	Problem identification	Determining important information in geometry problems	Questions 1–5: Determining the elements of a solid based on an interactive GeoGebra image	20
I2	Planning strategy	Designing resolution steps with the ATD approach	Questions 6–10: Formulating hypotheses about the relationships between sides and angles in three-dimensional shapes	20
I3	Implementation of techniques	Applying GeoGebra algorithms or constructs	Questions 11–15: Using GeoGebra to determine surface area and volume	20
I4	Interpretation of results	Interpreting calculation results in the context of real geometry	Questions 16–20: Analyzing the suitability of GeoGebra model results with theoretical conditions	20
I5	Evaluation and reflection	Evaluating errors and providing alternative solutions	Questions 21–25: Comparing two methods of solution based on GeoGebra results	20
Highest Total Score			100	

F. Data Analysis Techniques

Data analysis in this study was specifically conducted to examine the effect of GeoGebra-based geometry learning designed using the praxeology from the Anthropological Theory of the Didactic (ATD) on students' problem-solving skills. Pretest and posttest scores from both the experimental and control groups were analyzed descriptively to identify initial group equivalence and learning gains. Prior to hypothesis testing, data normality and variance homogeneity were examined to ensure the appropriateness of parametric analysis. An independent samples *t*-test was then applied to the posttest scores to determine whether significant differences existed between students who engaged in praxeology-based learning with GeoGebra and those who received praxeology-based conventional instruction without GeoGebra. In addition, paired samples *t*-tests were used to analyze within-group improvements, while effect size calculations were included to assess the magnitude and practical significance of the instructional intervention.

Table 5. Data analysis techniques and interpretation range

Types of Analysis	Interpretation Range
Construct Validity	Item > 0.5 (fit); *misfit* items revised or eliminated
Instrument Reliability	Alpha > 0.7 (consistent); *Person reliability* > 0.8
Normality Test	Sig > 0.05 (normal); Sig < 0.05 (non-normal)
Homogeneity Test	Sig > 0.05 (homogeneous); Sig < 0.05 (heterogeneous)
t-test (Independent)	Sig < 0.05 indicates a significant difference
N-Gain	>0.7 high; 0.3–0.7 moderate; <0.3 low
Relative Effectiveness	Effectiveness of the experimental class compared to the control class (d%)

Data analysis was conducted using quantitative inferential methods with the use of SPSS 28, JAMOVI, MINISTEP Rasch, and SmartPLS 4 software. Validity and reliability tests were performed using Corrected Item-Total Correlation and Cronbach's Alpha, while Rasch analysis was used to examine unidimensionality, item fit, and person reliability. Normality was tested using the Shapiro–Wilk test and homogeneity using Levene's test. The effectiveness of the learning model was analyzed using the Independent Samples t-Test, accompanied by N-Gain calculations to assess learning outcome improvement and effect size (Cohen's d) to measure the strength of influence. The analysis results were interpreted triangulatively based on statistical parameters and visual representations of the data. The statistical parameters referred to in each analysis technique are shown in Table 5 to determine the interpretation of the results.

IV. RESULT AND DISCUSSION

A. Results of Instrument Validity and Reliability Testing

The results in this subsection describe students' problem-solving performance after the implementation of GeoGebra-based geometry learning structured through the praxeology from ATD. The problem-solving test was developed to measure students' ability to understand problems, plan solution strategies, execute procedures, and interpret results, which correspond to the task-technique-technology components of praxeology.

An example of the implementation of the praxeology from ATD integrated with GeoGebra can be illustrated through a geometry learning task on the properties of quadrilaterals. Students were first presented with a task requiring them to identify invariant properties of a dynamically constructed quadrilateral. Using GeoGebra, students applied construction techniques such as dragging vertices to observe invariant relationships. The technological component of the praxeology was explicitly addressed through guided questions prompting students to justify why certain properties remained unchanged under transformation. This integration enabled students to move beyond procedural manipulation toward conceptual reasoning supported by dynamic visualization.

Descriptive analysis of pretest and posttest scores indicates an observable improvement in students' problem-solving performance in the experimental group compared to the control group. This improvement suggests that the learning design, which integrates mathematical tasks with GeoGebra-supported techniques and explicit technological justification, facilitated students' engagement with geometric problem situations in a more meaningful and structured manner. Rather than focusing on the technical process of instrument construction, this subsection emphasizes how the instrument outcomes reflect students' learning achievements aligned with the study's objective of enhancing problem-solving skills through praxeology-based instruction.

ENTRY NUMBER	TOTAL SCORE	TOTAL COUNT	JMLE MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD	PTMEASUR-AL CORR.	PTMEASUR-AL EXP.	EXACT MATCH OBS%	EXACT MATCH EXP%	Item
19	138	60	-.15	.21	1.31	1.66	1.30	1.63	A .38	.24	51.7	56.4	S19
3	130	60	.19	.21	1.25	1.34	1.25	1.35	B .07	.23	55.0	60.0	S3
13	141	60	-.28	.21	1.21	1.19	1.21	1.19	C .34	.24	48.3	55.2	S13
18	133	60	.06	.21	1.18	1.02	1.19	1.04	D .32	.24	50.0	58.7	S18
1	133	60	.06	.21	1.17	.98	1.17	.96	E .18	.24	53.3	58.7	S1
5	136	60	-.07	.21	1.15	.85	1.15	.84	F .33	.24	50.0	57.3	S5
22	126	60	.37	.21	1.13	.75	1.13	.75	G .25	.23	58.3	61.3	S22
21	133	60	.06	.21	1.11	.63	1.10	.62	H-.01	.24	60.0	58.7	S21
17	132	60	.11	.21	1.09	.53	1.09	.55	I .31	.24	51.7	59.1	S17
20	144	60	-.41	.21	1.08	.52	1.08	.52	J .01	.24	50.0	54.0	S20
14	128	60	.28	.21	1.07	.46	1.07	.46	K .14	.23	61.7	60.7	S14
25	135	60	-.02	.21	1.07	.43	1.06	.41	L .45	.24	58.3	57.8	S25
11	128	60	.28	.21	.97	-.13	.96	-.16	M .19	.23	60.0	60.7	S11
24	140	60	-.24	.21	.95	-.22	.95	-.20	l .20	.24	45.0	55.6	S24
7	125	60	.41	.21	.94	-.28	.94	-.30	k .05	.23	65.0	61.6	S7
2	142	60	-.32	.21	.93	-.37	.93	-.35	j .30	.24	51.7	54.8	S2
12	135	60	-.02	.21	.93	-.32	.93	-.35	i .08	.24	65.0	57.8	S12
23	132	60	.11	.21	.92	-.38	.92	-.38	h .14	.24	68.3	59.1	S23
10	134	60	.02	.21	.90	-.52	.90	-.51	g .28	.24	63.3	58.3	S10
15	137	60	-.11	.21	.85	-.86	.85	-.82	f .34	.24	63.3	56.9	S15
6	132	60	.11	.21	.82	-1.03	.82	-1.03	e .37	.24	60.0	59.1	S6
16	141	60	-.28	.21	.79	-1.25	.79	-1.27	d .23	.24	63.3	55.2	S16
4	137	60	-.11	.21	.75	-1.50	.75	-1.50	c .20	.24	61.7	56.9	S4
9	141	60	-.28	.21	.73	-1.64	.73	-1.64	b .18	.24	58.3	55.2	S9
8	129	60	.24	.21	.69	-1.87	.69	-1.86	a .50	.23	65.0	60.4	S8
MEAN	134.5	60.0	.00	.21	1.00	.00	1.00	.00			57.5	58.0	
P.SD	5.1	.0	.22	.00	.17	.96	.17	.96			6.3	2.1	

Fig. 3. Item fit statistics for Rasch model infit/outfit, point-measure correlation, and exact match.

Based on Fig. 3 (MINISTEP Rasch output; 25 items, N = 60), all items show adequate model fit. The MNSQ Infit range is approximately 0.75–1.31 with a ZSTD of approximately −2.11 to 1.31, while the MNSQ Outfit range is approximately 0.79–1.66 with a ZSTD of approximately −1.63 to 1.96. these values are within the practical feasibility corridor of 0.5–1.5 (quite strict) and are still tolerable up to ±2.0 for educational tests, so that no severe misfit was detected. The point-measure correlations (PTMEASUR-AL CORR.) are all positive with a range of approximately 0.14–0.38 (model expectation ~0.24), indicating that each item moves in the same direction as the latent ability measured and contributes to the consistency of the scale. The average exact match OBS of 57.5% is close to EXP 58.0%, indicating the suitability of the response pattern to Rasch predictions. The difficulty measure (MEASURE) distribution was relatively moderate (±≈1.4 logits; approximately −1.16 to + 0.28), indicating that the items were sufficiently targeted to distinguish respondents' abilities across a relevant range of competencies. One item with an Outfit MNSQ ≈1.66 shows irregular responses at a certain level of difficulty and warrants review of its wording/context, but does not interfere with the overall validity of the instrument. Overall, the evidence of item feasibility in Fig. 3 supports the construct validity and measurement accuracy of the geometry problem-solving instrument within the ATD-GeoGebra framework, making it suitable for further analysis (normality, homogeneity, t-test, N-gain, and effect estimation).

Based on Fig. 4, which displays the Wright Map resulting from Rasch model analysis, the distribution of respondent ability (person measure) and item difficulty (item measure) shows a good balance between participant characteristics and the instrument. On the vertical axis, the "X" mark represents the frequency of respondents, while the item codes are on the right side, indicating the difficulty level of each item. Most participants are distributed in the logit range between −1 and 0, indicating that the students' ability level is in the moderate category. On the other hand, the majority of items are located in the difficulty range of 0 to +1 logit, indicating that the instrument has adequate discriminating power and is able to challenge respondents with medium to high ability levels. Only a few items were below −1 logit, indicating that some questions were too easy for most respondents. This pattern shows that the distribution of items and persons is well calibrated, so that the measurement scale can reliably estimate problem-solving ability. There was no extreme overlap between item and respondent positions, indicating a match between item difficulty and student ability. Overall, this Wright Map shows the validity and balance of the ATD-GeoGebra-based geometry problem-solving ability instrument, meaning that the instrument items accurately and proportionally represent the variation in student abilities according to the Rasch measurement model.

```
MEASURE Item - MAP - Person
          <freq>||<more>
   1            ++
                ||
                ||
                ||
                ||
                ||
                ||
                ||T 02LK  23PE  25PE
   0            ++  08PE  10PE  12LK
                ||  15PE  22PE  28LK
                ||S 14LK  19LE  30LK
                ||  22PK  26LE
                ||  01LK  04LK  07LK  19PK  24PK  25LK  30LE
                ||  07PE  14LE  23LK  29PE
                ||M 05LE  06LK  21PK  28PE  29LK
                ||  02PE  08PK  09LK  11PE  17LE  18LE  18PK  26LK  27PE
             X T||  01PE  06PE  16PE  20LK  24LE  27PK
  -1      XXXX  ++S 03PK  09PE  10LK  12PE  13PE  20PE  21LE
            XX S||  15PK
         XXXXX  ||  04PE  05PK  11PK  13LK  16LK
          XXXX M||  03PE
          XXXX  ||T
           XXX S||  17PK
            XX  ||
               T||
                ||
  -2            ++
          <rare>||<less>
```

Fig. 4. Wright map (item–person map) Rasch model.

The performance of the instrument shows a very high level of measurement consistency, as reflected in Fig. 5, where the Person Reliability value reaches 0.84 (real) and 0.82 (model) with a person separation index of 3.57–3.68. This value indicates that the instrument has an excellent ability to distinguish students' ability levels into at least four strata of problem-solving competence. A reliability index above 0.8 confirms the internal stability of the instrument in producing consistent scores across respondents and shows that the variation in scores obtained is caused more by differences in actual ability than by measurement errors. The average Infit MNSQ (1.00) and Outfit MNSQ (1.04) indicate a very ideal model fit with the empirical data distribution, while the ZSTD value (±0.3–0.4) shows that there are no extreme deviations in the students' response patterns. The variation in total scores between 47 and 64 points indicates a proportional and well-measured range of abilities. With a standard error of measurement (SEM) = 0.32, this instrument shows adequate precision for experimental research applications. These results prove that the ATD-GeoGebra-based geometry problem-solving measurement tool has strong construct reliability, high discriminatory power, and stable measurement model consistency, making it suitable for testing the effectiveness of learning models and further analysis using an inferential quantitative approach.

	TOTAL SCORE	COUNT	MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD
MEAN	56.0	25.0	-.65	.32	1.00	-.01	1.00	-.01
SEM	.5	.0	.05	.00	.04	.13	.04	.13
P.SD	3.8	.0	.39	.00	.28	1.03	.28	1.03
S.SD	3.8	.0	.39	.00	.29	1.04	.29	1.04
MAX.	64.0	25.0	.16	.33	1.83	2.49	1.82	2.48
MIN.	47.0	25.0	-1.60	.32	.28	-3.58	.28	-3.59

REAL RMSE .34 TRUE SD 1.19 SEPARATION 3.57 Person RELIABILITY .84
MODEL RMSE .32 TRUE SD 1.22 SEPARATION 3.68 Person RELIABILITY .82
S.E. OF Person MEAN = .15

Fig. 5. Reliability and separation statistics of respondents (person reliability and separation index) Rasch model.

The visualization in Fig. 6 shows the item fit distribution against the Rasch model based on the Outfit ZSTD value. All items in the instrument are within the tolerance range of −2 to +2, which is the ideal zone for showing model fit (fit region). The distribution of items concentrated in the middle area with ZSTD values close to 0 shows that the students' response patterns to all items are consistent, stable, and do not contain extreme anomalies that indicate overfitting or underfitting. The relatively close position of the items also shows the homogeneity of the level of difficulty and the balance between the respondents' abilities and the characteristics of the questions. No items were found outside the extreme yellow area, confirming that there were no items with significant deviations from the Rasch probabilistic model expectations. This distribution pattern illustrates that the instrument has high construct validity, where each item contributes proportionally to the measurement of students' geometric problem-solving abilities. The density between items in the middle area reinforces the interpretation that the instrument items have good internal stability and that there is no measurement bias between respondents. Thus, this figure provides empirical evidence that the Rasch measurement model in this study is statistically and theoretically fit, and supports the use of the instrument to assess the effectiveness of geometry learning based on the Anthropological Theory of the Didactic (ATD) with the integration of GeoGebra in a valid and reliable manner.

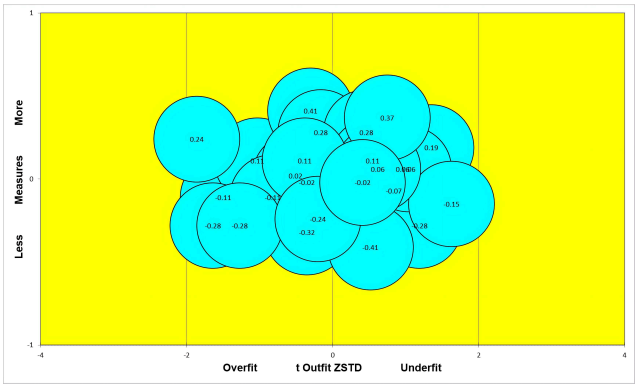

Fig. 6. Map of item suitability distribution based on ZSTD outfit values in the Rasch model.

B. Results of Normality and Homogeneity of Variance Tests

To determine whether the observed improvement in problem-solving skills was statistically significant, inferential analysis was conducted based on the research design and data characteristics. The selected statistical tests were used to compare the problem-solving performance of students in the experimental and control groups after the intervention.

The results of the analysis demonstrate a significant difference in posttest scores between the two groups, indicating that students who learned geometry using GeoGebra within the praxeology from ATD framework achieved higher problem-solving performance than those who experienced conventional instruction. These findings confirm that the implemented learning design had a measurable effect on students' problem-solving abilities. Accordingly, this subsection focuses on how the statistical results support the research purpose of evaluating the effectiveness of the GeoGebra-based, praxeology-oriented learning approach, rather than merely justifying the choice of statistical tests.

The normality test shows that the distribution of scores meets the parametric assumptions for all combinations of class and measurement time. With a sample size per group of $n = 30$, Shapiro–Wilk is prioritized because it is stronger for small to medium samples. The Shapiro–Wilk significance values were 0.131 (Experimental Pretest), 0.182 (Control Posttest), 0.913 (Experimental Posttest), and 0.502 (Control Pretest), all of which were > 0.05, so the distribution could be declared normal. The Kolmogorov–Smirnov test with Lilliefors correction indicated one cell $p = 0.028$ (Experimental Pretest) as an indication of slight deviation; however, this inconsistency is common in Likert-based total score data and, in accordance with methodological practice, the final decision refers to Shapiro–Wilk, which is more sensitive in this n range. Consequently, further analysis using parametric procedures (t-test, effect size calculation, and N-Gain based on the assumption of normality which is presented in Table 6 below) is warranted.

Table 6. Normality test results (Kolmogorov–Smirnov and Shapiro–Wilk)

Variabel	Class	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Results	Pretest Exp	0.169	30	0.028	0.946	30	0.131
	Posttest Cotr	0.145	30	0.106	0.951	30	0.182
	Pretest Exp	0.115	30	0.200*	0.984	30	0.913
	Posttets Cotr	0.131	30	0.200*	0.969	30	0.502

Note: *. This is a lower bound of the true significance.
a. Lilliefors Significance Correction

The homogeneity of variance test presented in Table 7 shows that the variance between research groups is balanced and meets the parametric assumption. The significance values for all test approaches based on the mean (Sig. = 0.216), median (Sig. = 0.303), median with adjusted df (Sig. = 0.304), and trimmed mean (Sig. = 0.218) are all greater than $\alpha = 0.05$, so it can be concluded that there are no significant differences in variance between groups. The consistency of the significance values above the critical limit indicates that the data has good variance stability between the experimental and control classes. These results show that the distribution of students' problem-solving abilities in both groups is homogeneous, so that any differences in learning outcomes that arise can be attributed to the learning treatment rather than to differences in variability between groups. This condition of homogeneity strengthens the inferential validity of the study and ensures that the independent t-test analysis and calculation of the effectiveness of the ATD-GeoGebra-based learning model can be carried out with a valid and reliable statistical basis.

Table 7. Results of the homogeneity of variance test (Levene's test of equality of variances)

Variabel	Test Method	Levene Statistic	df1	df2	Sig.
Results	Based on Mean	1.507	3	116	0.216
	Based on Median	1.227	3	116	0.303
	Based on Median and with adjusted df	1.227	3	107.807	0.304
	Based on trimmed mean	1.503	3	116	0.218

C. *Pretest and Posttest Results*

The analysis of pretest and posttest scores indicates a significant improvement in students' problem-solving skills in the experimental group compared to the control group. However, to provide a more comprehensive understanding of the learning process, these quantitative findings are supported by qualitative data derived from classroom observations and field notes collected during the intervention.

Classroom observations revealed that students in the experimental group demonstrated higher engagement when interacting with GeoGebra-based tasks. During learning activities, students were more actively involved in exploring geometric relationships, formulating conjectures, and testing solutions dynamically. Field notes documented frequent instances of students revising their strategies after visual feedback from GeoGebra, indicating the emergence of reflective problem-solving behaviors aligned with the praxeology components of task, technique, and technology.

In contrast, observations in the control group showed that students relied primarily on procedural techniques presented in textbooks, with limited exploration and fewer opportunities to revise their solutions. Field notes indicated that problem-solving activities in the control group were largely teacher-directed, resulting in minimal student-initiated reasoning and reduced interaction among peers.

These qualitative findings corroborate the quantitative results, suggesting that the integration of GeoGebra within the praxeology from the Anthropological Theory of the Didactic facilitates not only higher posttest scores but also deeper cognitive engagement in problem-solving processes. Thus, the improvement observed in the experimental group reflects both measurable learning outcomes and meaningful changes in students' learning behaviors.

The achievement pattern shows a strong improvement in performance in the experimental class and a moderate improvement in the control class, consistent across all indicators I1–I5 (see Table 8). The total average for the experimental class rose by 30.2 points from 56.3 (SD = 3.73) in the pretest to 86.5 (SD = 2.69) in the posttest; the score range shifted from 49–64 to 82–92, accompanied by a decrease in standard deviation, indicating a more uniform distribution of scores after the ATD + GeoGebra intervention. The increase in the inter-class coefficient was relatively balanced (I1: +5.8; I2: +6.0; I3: +5.9; I4: +5.9; I5: +6.6), with a post-test median of 17–18, confirming the consistency of the increase in the distribution center. In the control class, the total average increased by 19.1 points from 55.8 (SD = 3.94) to 74.9 (SD = 3.29); the range shifted from 47–64 to 67–83, and a reduction in variance was also apparent but not as sharp as in the experiment. Per indicator, the control increase was in the range of +3.9 to +4.3, lower than the experiment. The convergence of the control median at 15 for all post-test indicators suggests a stable but less powerful ATD learning effect without GeoGebra than the dynamic integration in the experimental group. Substantively, these descriptive findings position GeoGebra integration within the ATD framework as an intervention that not only increases average scores but also refines the distribution of problem-solving abilities—an important indication that dynamic representation-guided praxeological practices strengthen both conceptual and procedural understanding in students.

Table 8. Descriptive pretest–posttest by indicator

Variable	Group	Test	I1	I2	I3	I4	I5	Total
N	Exper.	Posttest	30	30	30	30	30	30
		Pretest	30	30	30	30	30	30
	Control	Posttest	30	30	30	30	30	30
		Pretest	30	30	30	30	30	30
Mean	Exper.	Posttest	17.4	17.3	17.1	17.4	17.3	86.5
		Pretest	11.6	11.3	11.2	11.5	10.7	56.3
	Control	Posttest	14.9	15.1	14.9	15.0	15.1	74.9
		Pretest	11.0	10.8	11.1	11.5	11.5	55.8
Median	Exper.	Posttest	17.0	17.5	17.0	18.0	17.5	86.0
		Pretest	12.0	11.0	11.0	11.0	11.0	55.0
	Control	Posttest	15.0	15.0	15.0	15.0	15.0	75.5
		Pretest	11.0	11.0	11.0	11.0	12.0	56.0
SD	Exper.	Posttest	1.35	0.99	1.24	1.43	1.09	2.69
		Pretest	1.43	1.08	1.30	1.33	1.26	3.73
	Control	Posttest	1.07	1.27	1.25	1.08	1.28	3.29
		Pretest	1.34	2.05	1.68	1.25	1.59	3.94
Min.	Exper.	Posttest	14	16	14	15	15	82
		Pretest	9	10	9	9	9	49
	Control	Posttest	13	13	12	12	13	67
		Pretest	9	5	8	9	8	47
Max.	Exper.	Posttest	20	19	19	20	20	92
		Pretest	14	14	14	14	14	64
	Control	Posttest	16	19	17	17	18	83
		Pretest	14	15	15	14	15	64

D. *Independent Samples t-Test Results*

The Independent Samples t-Test presented in Table 9 shows a significant difference between the experimental and control classes in all problem-solving ability indicators (I1–I5). The t-values range from 6.94 to 7.93 with a significance level of $p < 0.001$, confirming that the mean difference between the two groups is statistically significant. The relatively high mean difference, ranging from 2.23 to 2.50 points, indicates that the integration of GeoGebra in geometry learning based on the Anthropological Theory of the Didactic (ATD) has a substantive impact on improving students' ability to identify problems, design strategies, apply techniques, interpret results, and conduct reflective evaluations. The low standard error difference values (0.294–0.327) reinforce the reliability of the mean difference estimates, indicating stability in variance between groups. Although there was a violation of homogeneity (Levene's test significant) in indicator I4, the t-test results remained valid because the p-value was < 0.001 and the mean difference showed a consistent direction of influence. Theoretically, these findings confirm that ATD–GeoGebra learning is more effective than conventional ATD in developing higher-level mathematical problem-solving skills, in line with the ATD assumption about the role of technology as an epistemic component that strengthens the relationship between task-technique-technology-theory in the context of geometry learning.

Table 9. Results of the independent samples t-test per problem-solving ability indicator

Indicator	Participant	Statistic	df	p	Mean difference	SE difference
I1	Student's t	7.93	58.0	<0.001	2.50	0.315
I2	Student's t	7.59	58.0	<0.001	2.23	0.294
I3	Student's t	6.94	58.0	<0.001	2.23	0.322
I4	Student's t	7.33[a]	58.0	<0.001	2.40	0.327
I5	Student's t	7.36	58.0	<0.001	2.27	0.308

Note: H_a $\mu_{Eksperimen} \neq \mu_{Kontrol}$

[a] Levene's test is significant ($p < 0.05$), suggesting a violation of the assumption of equal variances

E. N-Gain Test Results

The N-Gain analysis presented in Table 10 shows a striking difference in the level of improvement in problem-solving skills between the experimental group and the control group.

Table 10. Descriptive statistics of students' problem-solving ability n-gain

Group			Statistic	Std. Error
Experiment	Mean		69.02	1.219
	95% Confidence Interval for Mean	Lower Bound	66.52	
		Upper Bound	71.51	
	5% Trimmed Mean		69.23	
	Median		68.76	
	Variance		44.577	
	Std. Deviation		6.677	
	Minimum		53	
	Maximum		80	
	Range		27	
	Interquartile Range		11	
	Skewness		–0.217	0.427
	Kurtosis		–0.146	0.833
Control	Mean		42.96	1.522
	95% Confidence Interval for Mean	Lower Bound	39.84	
		Upper Bound	46.07	
	5% Trimmed Mean		43.30	
	Median		42.86	
	Variance		69.511	
	Std. Deviation		8.337	
	Minimum		18	
	Maximum		60	
	Range		42	
	Interquartile Range		9	
	Skewness		–0.795	0.427
	Kurtosis		1.893	0.833

The average N-Gain value for the experimental class was 69.02% with a standard deviation of 6.677, indicating an improvement in the high category ($0.7 > g \geq 0.3$) according to Hake's classification. The score range between 53 and 80 with a median of 68.76 and a 95% confidence interval (66.52–71.51) shows the stability of the increase in student learning outcomes across all ability indicators. The relatively symmetrical data distribution (skewness = –0.217; kurtosis = –0.146) reflects a normal and homogeneous distribution among participants. In contrast, the control group showed an N-Gain average of only 42.96% with a standard deviation of 8.337, which is in the moderate category, with a wider range (18–60) and greater variance (69.511), indicating inconsistency in improvement between individuals. The mean difference of 26.06 points between the two groups proves that the application of the ATD learning model integrated with GeoGebra results in higher and more stable effectiveness gains than conventional ATD learning. These results reinforce the findings of the previous t-test that the use of GeoGebra not only improves final scores but also maximizes student learning gains through dynamic visualization experiences, praxeological exploration, and deeper conceptual reflection in the context of geometry learning based on the Anthropological Theory of the Didactic (ATD).

F. Discussion

The increase in the average post-test scores and the wide N-Gain gap between the experimental and control classes confirm that the integration of GeoGebra into the ATD praxeological framework does not merely accelerate procedures, but enriches the conceptual reasoning that supports each step of problem solving (I1–I5). The stability of the improvement in the experimental class reflected in the smaller variance indicates a didactic orchestration that consistently links task-technique-technology-theory throughout the learning cycle. The lower variance observed in the experimental class is not merely a statistical indication of score consistency but reflects the completion of a didactic learning cycle grounded in praxeology from the ATD. During the implementation phase, students repeatedly engaged with tasks supported by GeoGebra-based techniques, which enabled them to test, revise, and validate mathematical technologies. The reflection phase occurred when students interpreted their problem-solving outcomes through guided discussions and feedback, allowing the alignment of techniques with underlying theoretical justifications. This iterative process strengthened the coherence between task–technique–technology–theory, resulting in more stable learning outcomes across students.

This pattern is consistent with evidence that dynamic visual ecosystems facilitate a shift from symbolic manipulation to testable conceptual argumentation, thereby strengthening the reliability of performance across indicators [22, 47]. These findings support the argument that geometry-based dynamic media enhance representational coherence and narrow the gap between visual models and theoretical structures, a prerequisite for advanced problem-solving achievements in higher education.

The significant advantages of the experimental class across all indicators show that the GeoGebra-assisted didactic situation design activates a layered cognitive pathway: identification of essential information (I1), strategy planning (I2), technique implementation (I3), interpretation of results (I4), and evaluation-reflection (I5). In particular, the surge in I2 and I4 indicates that the dynamic interface enables students to test local hypotheses, visualize dependencies between objects, and assess the consistency of results with relevant axioms and theorems. Similar evidence emerges in studies documenting increased conceptual confidence and strategic persistence when calculus/geometry is taught with GeoGebra support [26, 27]. On the other hand, the strengthening of I5 confirms the role of re-representation as a reflective mechanism for isolating sources of error and synthesizing alternative solutions, in line with reports of increased diagnostic accuracy when visual exploration is combined with deductive reasoning [15, 16].

The superior performance of the experimental class shows that ATD functions as a binding architecture that ensures every GeoGebra activity is rooted in explicit epistemic goals. By mapping tasks to accountable techniques, formulating technological justifications (reasons why techniques work), and referring to legitimate theories, learning moves from "drawing correctly" to "thinking correctly" about geometric objects. This approach reduces dependence on ad-hoc procedures and improves the transfer of strategies to new problems. Praxeology literature asserts that consistency in educators' documentation of didactic work including the selection of digital artifacts correlates with the quality of orchestration and stable learning outcomes [48, 49]. The findings of this study extend this evidence by showing that praxeology enriched with dynamic artifacts produces measurable learning gains across all problem-solving indicators.

Adequate normality and homogeneity of variance between groups, accompanied by smaller N-Gain variance in the experimental class, indicate that the ATD GeoGebra-based didactic design effectively mitigates learning obstacles that are common in spatial geometry topics (misinterpretation of angle/side relationships, misconnections between representations, and difficulties in generalization). Mapping obstacles into a tiered task design from guided exploration to formal justification is in line with the diagnosis-intervention approach recommended by recent research on didactic phenomena and learning obstacles [50]. In addition, the arrangement of tasks that forces students to externalize the technological reasons behind GeoGebra techniques minimizes procedural error variation, resulting in a more uniform distribution of achievement without sacrificing conceptual depth [51].

Consistent positive effects across all indicators are intertwined with affective-motivational dimensions that often mediate strategy persistence and reflection quality. A dynamic environment that allows for immediate feedback and what-if analysis triggers task engagement, reduces extrinsic cognitive load, and strengthens academic self-efficacy factors that have been shown to improve the success of planning and revising strategies when dealing with non-routine problems [52, 53]. Within this framework, ATD provides epistemic cues so that interest does not stop at visual manipulation, but is directed towards conceptual justification and theoretical reasoning. It is this synergy between increased engagement and epistemic control that explains why learning gains are not only higher but also more stable in the experimental class.

V. Conclusion

Reconstructing geometry learning through GeoGebra by explicitly employing praxeology from the Anthropological Theory of the Didactic (ATD) provides a meaningful contribution to enhancing students' problem-solving skills. By organizing learning activities around tasks, techniques, technologies, and underlying theories, GeoGebra functioned not merely as a visualization tool but as a didactic medium that supported students' systematic exploration and reasoning in geometry. The empirical findings indicate that students who engaged in GeoGebra-based praxeological learning demonstrated significantly better problem-solving performance than those who experienced conventional instruction. These results suggest that digital environments, when grounded in a clearly defined praxeological structure, can promote deeper conceptual understanding and strategic mathematical thinking. Theoretically, this study clarifies that the instructional design is informed solely by praxeology from ATD, rather than the full ATD framework, thereby ensuring conceptual precision. Practically, the proposed learning design offers an alternative instructional model for geometry teaching in higher education. Future research may investigate the application of this approach in other mathematical topics and examine its sustainability across different learning contexts.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Nanang Diana served as the lead author who designed the research, formulated the theoretical framework based on Anthropological Theory of the Didactic (ATD), and led the integration of GeoGebra in the geometry learning design. Agustinasari was responsible for quantitative data analysis using SPSS, JAMOVI, MINISTEP Rasch, and SmartPLS 4 software, as well as interpreting inferential statistical results and the effectiveness of the learning model. Husnul Khatimah contributed to the development of problem-solving ability test instruments, content and construct validation of instruments, and the preparation of cognitive indicators based on Polya's theory and ATD. Adi Apriadi Adiansha led the compilation and alignment of the literature review with the latest research, performed scientific editing of the final manuscript, and ensured the writing style was in accordance with reputable international publication standards. M. Ibnusaputra played a role in collecting empirical data, visualizing the research results through tables and graphs, and verifying the reliability of the experimental procedures. All authors had approved the final version.

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