Student Cognitive Assessment to Fraction Learning Pathways through Artificial Intelligence

Eleni Lekati *, Konstantinos Lazaros, Aristidis Vrahatis, Panagiotis Vlamos, and Spyridon Doukakis

Department of Informatics, Ionian University, Corfu, Greece

Email: elenlekati@ionio.gr (E.L.); Lakonstant@ionio.gr (K.L.); aris.vrahatis@ionio.gr (A.V.); vlamos@ionio.gr (P.V.); sdoukakis@ionio.gr (S.D.)

*Corresponding author

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Abstract—**This study investigates how technology-enhanced tools and Artificial Intelligence (AI) can be used to personalize fraction learning in primary education through cognitive profiling. Fractions are a foundational area of mathematics, yet many students struggle to understand their abstract concepts. While research has shown that cognitive skills such as working memory, attention, and visual perception influence mathematics performance, there is limited evidence on how these factors specifically shape students' Conceptual Understanding (CU) and Procedural Knowledge (PK) of fractions. To address this gap, senior primary school students in Greece were assessed using the RODI cognitive profiling application, the Fraction Lab learning tool, and additional validated measures of CU and PK. AI-based analyses were applied to examine relationships between cognitive skills, fraction performance, and learning outcomes, and to map students into distinct cognitive profiles. Results demonstrated strong links between visual perception, executive function, and fraction performance. Distinct profiles revealed different patterns of strength and challenge, which were then used to design adaptive learning pathways. The findings highlight how digital tools can provide teachers with actionable insights for tailoring instruction, enhancing engagement, and improving outcomes in fraction learning.**

Keywords—**fraction learning, primary education, cognitive profiling, digital assessment tools, artificial intelligence**

I. Introduction

Personalization in education has become an increasingly important approach for meeting the diverse needs of learners. Unlike traditional, one-size-fits-all methods, personalized education focuses on adapting teaching strategies and content to the specific cognitive, emotional, and developmental characteristics of each student. This approach recognizes that schools play a vital role not only in transmitting knowledge but also in supporting students' confidence, emotional well-being, and ability to become lifelong learners. By tailoring instruction to individual needs, it is possible to address learning difficulties more effectively, particularly in mathematics, while also enhancing student motivation and engagement. Mathematics at the primary school level is a key area for implementing personalized approaches. As a core subject, mathematics develops essential problem-solving and analytical skills that are fundamental to academic success and everyday life [1, 2]. Within mathematics, fractions represent a critical topic. They serve as a foundation for more advanced mathematical areas such as algebra and geometry, while also influencing scientific reasoning and daily decision-making [3].

Nevertheless, many students struggle to understand fractions because of their abstract nature and the high cognitive demands they impose [4]. Common challenges include understanding fraction equivalence, ordering fractions, and performing operations, tasks that require advanced cognitive processes such as visual perception, working memory, and attention [5, 6]. Previous research has shown a strong relationship between general cognitive skills and mathematics achievement, with about half of the variance in mathematics performance explained by these skills [7, 8]. Cognitive abilities such as visual perception, working memory, and attention are therefore important predictors of how well students learn mathematical concepts [8, 9]. These skills are especially critical during transitional stages in education, such as the move from primary to secondary school, when students face increasingly complex mathematical tasks [10–12].

Visual perception influences how students interpret diagrams, number lines, and area models used to represent fractions. Students with strong visual perception skills are more likely to identify proportional relationships and compare fractions accurately [13]. Working memory enables students to hold and manipulate information while solving multi-step fraction problems, such as adding fractions with unlike denominators [14, 15]. Attention helps students stay focused on key elements of a problem (e.g., numerators and denominators) while ignoring distractions and managing cognitive load during complex tasks [16, 17].

Given these difficulties, personalized learning offers a way to adapt instruction to students' cognitive strengths and weaknesses [18]. The Aptitude-Treatment Interaction (ATI) model is a well-established framework that matches teaching strategies with students' cognitive profiles. By grouping students according to their skill levels—high, medium, or low—the ATI model ensures that each learner receives the right level of guidance. Students with higher cognitive abilities can engage in more independent tasks, while those with lower abilities benefit from more structured support [19]. The ATI model enables the development of customized learning experiences, enhancing student understanding and engagement, particularly in challenging subjects like fractions [20, 21]. Tools such as RODI [22], Fraction Lab and research frameworks proposed by Bempeni *et al.* [23, 24] have offered promising methodologies for addressing these challenges, yet they often overlook the need for deeper personalization in learning strategies when standalone.

Fraction learning poses distinct cognitive challenges that extend beyond general mathematical difficulty, as it requires the coordinated use of visual-spatial processing, working memory, and attentional control. Students must interpret visual representations such as number lines and area models,

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maintain intermediate values during multi-step procedures, and selectively attend to relevant numerical relationships while suppressing misleading whole-number intuitions. While prior research has established the importance of these cognitive abilities for fraction learning, such work has often examined cognitive factors in isolation or relied on static assessment methods that offer limited insight into how these abilities manifest during authentic learning activities. Cognitive profiling therefore provides a principled way to capture learner-specific constraints and explain why students with similar instructional exposure may follow markedly different learning trajectories. However, cognitive profiles alone are insufficient to inform instructional design without contextualized evidence of how students engage with fraction tasks. Digital learning environments address this limitation by generating fine-grained data on students' interactions, strategies, and errors during learning. Artificial intelligence is then required to integrate cognitive indicators with behavioral performance data, enabling the identification of meaningful patterns and individualized learning pathways that cannot be reliably detected through traditional assessment or digital performance measures when considered in isolation.

In this context, the present study examines how technology-enhanced tools can be combined with cognitive profiling methods to deliver truly personalized instruction in fraction learning. The approach relies on two complementary forms of student profiling and two interactive digital tests—RODI and Fraction Lab—that generate detailed performance data. By analyzing these data through AI-driven models, the study seeks to uncover distinct learning pathways that align with each student's cognitive profile, highlighting both their strengths and the areas where they require additional support. This dual assessment system not only allows the identification of meaningful patterns in how students engage with fractions but also enables the design of targeted instructional routes that adapt to different learner needs. Through this integration of cognitive analytics and adaptive digital environments, the study demonstrates how technology can offer teachers actionable insights, refine instructional decision-making, and ultimately enhance both conceptual understanding and procedural fluency in fraction learning.

The contribution of this study warrants explicit articulation in relation to the existing body of research on fraction learning and cognitive assessment. While the individual components of our methodological approach—RODI for cognitive profiling, Fraction Lab for digital fraction assessment, and validated measures of conceptual and procedural knowledge—are grounded in established research traditions, the novelty resides in their systematic integration through interpretable machine learning to identify individualized learning pathways. Previous investigations have typically examined cognitive predictors of mathematical achievement or implemented digital environments for fraction instruction as separate lines of inquiry, thereby limiting the capacity to inform precision-oriented instructional design. The present study addresses this limitation through a dual-profiling methodology that synthesizes domain-general cognitive characteristics with domain-specific performance data captured during interactive fraction tasks. This synthesis is made possible through AI-driven analysis that reveals patterns of cognitive-behavioral interaction which remain inaccessible under conventional analytical frameworks. The primary contribution is therefore methodological in nature: demonstrating how the convergence of cognitive profiling, digital learning environments, and transparent machine learning can generate empirically grounded, actionable pathways for differentiated instruction. What distinguishes this work is not the isolated application of these tools, but rather their coordinated deployment to achieve a form of technology-enhanced personalization that bridges cognitive science and classroom practice, moving beyond descriptive assessment toward a framework that directly informs adaptive pedagogical decision-making in fraction learning.

A. Theoretical Framework and Research Questions

Building on the distinction between conceptual and procedural knowledge, Conceptual Understanding (CU) is often defined as a deep comprehension of the fundamental concepts and principles within a particular domain [25]. In mathematics, and particularly in the context of fractions, CU involves grasping the relationship between the numerator and the denominator, understanding part-whole interpretations, recognizing equivalence, and comparing magnitudes [26, 27]. It also includes an awareness of the rationale behind mathematical procedures, enabling learners to make sense of the operations they perform [28].

In contrast, Procedural Knowledge (PK) refers to the ability to accurately carry out fraction operations, focusing on the sequence of steps needed to complete mathematical tasks [25, 26, 28, 29]. The procedural knowledge associated with fractions is particularly complex, as different operations demand distinct computational approaches. For example, multiplying fractions requires processing the numerators and denominators separately, a strategy that differs from the methods used for addition or subtraction.

The distinction between CU and PK is well documented in psychology and mathematics education research [30–32]. Although they are conceptually distinct, research indicates that CU and PK are interdependent, with the development of one reinforcing the other [30]. Traditional instruction often emphasizes one dimension more than the other; however, approaches that integrate both CU and PK tend to promote deeper learning experiences [33]. Moreover, theories prioritizing conceptual understanding argue that students initially develop CU, which subsequently facilitates the acquisition and refinement of PK through repeated practice [29].

A recent systematic literature review [6] identified a broad consensus among researchers regarding the distinct developmental trajectories of CU and PK in the context of fractions. Additionally, the review highlighted a growing trend toward more targeted investigations, emphasizing the analysis of students' cognitive profiles alongside the nuanced characteristics of fractions under study. This growing focus on individualized learning pathways reflects an urgent need for approaches that can bridge these developmental gaps more effectively. Such efforts naturally align with emerging technological innovations, which hold promise for addressing these challenges in novel ways. Against this background, the present study aims to systematically examine the relationships between students' cognitive abilities and their conceptual and procedural performance in fraction learning, and to identify individualized learning pathways through the integration of cognitive profiling, digital learning environments, and artificial intelligence. Guided by these ideas, the present study focuses on the following research questions:

- To what extent have 6th-grade students in the Greek education system developed both Procedural Knowledge (PK) and Conceptual Understanding (CU) of fractions, as assessed through digital tools and personalized evaluation methods?
- What are the specific relationships between cognitive abilities—visual perception, working memory, and attention—and students' performance in conceptual and procedural fraction tasks, and how do these relationships inform the development of cognitive profiles?
- How can the integration of cognitive profiling and Artificial Intelligence (AI) analysis facilitate the identification of learning patterns, predict fraction learning outcomes, and support the design of individualized educational pathways in fraction comprehension?

By addressing these questions, this study aims to demonstrate how the combination of cognitive profiling, AI analysis, and adaptive digital tools can provide actionable insights for teachers, helping them personalize instruction and optimize student learning in real classroom settings, using readily available digital assessments and automated analytics that can be embedded into routine teaching practices without substantial changes to curriculum structure or specialized technical training**.**

B. Related Work

Numerous studies have investigated the use of technologies to address the difficulties students face in learning fractions. Specifically, Hwang *et al.* [34] developed a learning system based on multi-touch tabletops to facilitate collaborative fraction learning among elementary school students. The results indicated that using these surfaces enhanced student collaboration, improved understanding of fraction concepts, and developed their problem-solving skills.

According to Shin *et al.* [35], the use of virtual manipulatives for teaching fraction equivalence helped students focus on mathematical processes and the relationships between equivalent fractions. This approach enabled students to analyze fraction-related problems more thoroughly and efficiently. Digital educational games have also shown positive effects on students' understanding of fractions [36]. Games such as Refraction [37], Pizza al Lancio [38], Semideus [39], Motion Math, and Slice Fractions [40] were specifically designed to address difficulties in teaching rational numbers by applying best practices in mathematics education. Developed either by researchers or in collaboration with schools, these games focus on developing conceptual understanding while increasing student engagement and motivation.

Mohamed *et al.* [41] found that the use of multiple representations—such as real-world situations, manipulative aids, pictures, and symbols—enhances students' understanding of fractions. However, they observed that in the early stages of teaching fractions in elementary school, less emphasis is placed on developing learning through these representations. In another study [34], the Ubiquitous Fraction (U-Fraction) application was developed to support fraction learning through authentic contexts. U-Fraction allows students to take photos of objects from their environment and use them to create fraction representations in verbal, symbolic, and visual forms. This approach facilitates students' knowledge construction, as supported by Refs. [42, 43]. Such tools demonstrate the value of integrating authentic contexts and multiple representations into technology-enhanced learning environments, particularly for foundational topics like fractions.

In this context, innovative tools from neuroscience and Artificial Intelligence (AI) have the potential to significantly extend the use of educational technology. These tools make it possible to explore students' brain activity as they engage with fraction concepts, providing insights into the cognitive processes that underpin learning. AI algorithms can then analyze these complex data to reveal patterns and relationships that inform personalized teaching strategies [44, 45]. Although this type of integration is still at an early stage, its potential to combine cognitive insights with adaptive technologies makes it a promising direction for improving both Conceptual Understanding (CU) and Procedural Knowledge (PK) of fractions.

This study builds on these advancements by examining the cognitive and learning profiles of 6th-grade students in Greece, focusing on their CU and PK of fractions. Established assessment tools and AI-based methodologies were used to evaluate students' performance. Cognitive profiles were analyzed alongside learning outcomes to identify key factors, correlations, and adaptive learning pathways. Student cognitive profiles were derived from two sources: (a) performance in CU and PK tasks related to fractions, and (b) domain-general cognitive functions such as working memory and cognitive control, assessed using the RODI app. The integration of these dimensions provided a more detailed understanding of learners' strengths and areas for support, helping to identify personalized learning pathways.

By combining brain activity measurements, AI-driven analysis, and digital assessment tools, this study offers a comprehensive framework for tailoring educational strategies to the needs of individual students. The findings aim to show how classroom teachers can integrate such tools to better address cognitive differences and enhance fraction learning. Given the demonstrated effectiveness of personalized learning in improving academic outcomes, it is important to further explore its impact on students' understanding of fractions. Research has shown that diagnostic assessments can improve mathematics performance by 22% [46], while AI-based learning platforms can increase problem-solving skills by 30% and conceptual understanding by 25% [47]. Moreover, personalized learning can help reduce the achievement gap by up to 40%, supporting greater equity in education. Building on these findings, the present study integrates a range of research tools to examine student profiles based on cognitive skills and their levels of CU and PK in fractions [48].

II. Methodology

This study investigates the conceptual and procedural understanding of fractional numbers among sixth-grade students nearing the end of primary education in the Greek school system. At this stage, students are expected to have developed foundational knowledge of fractions, integrating both core conceptual understanding and procedural competence. To examine this, the study uses technology-enhanced tools specifically designed to support fraction learning, allowing a detailed evaluation of how students interact with digital tasks. Their performance across these environments was analyzed to identify patterns of strong, moderate, or limited understanding. The primary objective is to assess the extent to which Conceptual Understanding (CU) and Procedural Knowledge (PK) of fractions have been acquired by the completion of primary education, offering a clearer picture of students' mathematical cognition at this critical point. The data produced by these tools were analyzed using advanced statistical methods and machine learning techniques. Through this analysis, the study explored how cognitive performance relates to task outcomes, identified individualized learning pathways, and mapped the connections between students' cognitive profiles and their success in fraction-related activities.

Artificial Intelligence (AI) tools were incorporated to enrich this process by providing a deeper analysis of the relationship between cognitive profiles and students' conceptual and procedural performance. Traditional paper-based assessments often fail to capture the complexity of how students think and solve mathematical problems, limiting the ability to tailor instruction. By contrast, the AI-driven analysis applied in this study—based on machine learning models—allows the detection of subtle patterns, correlations, and individualized learning routes that would be difficult to identify manually. These models use performance data to highlight students' cognitive strengths and weaknesses, enabling the dynamic personalization of instructional content. This integration enhances our understanding of how cognitive factors shape fraction learning and equips educators with data-informed insights for more targeted and effective intervention strategies.

To ensure methodological appropriateness and interpretability given the limited sample size (n = 29), the AI-driven analysis relied on low-complexity, transparent machine learning techniques rather than data-intensive predictive models. Decision tree classifiers were selected because they perform reliably with small datasets, require minimal assumptions about data distributions, and produce explicit decision rules that can be readily interpreted in educational contexts. Input features consisted of aggregate, domain-level scores derived from validated instruments, including RODI cognitive indicators (Visual Category, Learning and Memory, Attention/Executive Function) and performance scores from the Procedural and Conceptual Understanding of Fractions Tool. All features were normalized prior to analysis to ensure comparability across scales. Model training focused on predicting categorical performance levels (low, medium, high), defined using normalized score thresholds applied to the 0–1 scale for interpretability and reproducibility (see Section 2.4 Model Specification and Evaluation for exact cutoffs). Tree depth and minimum sample splits were constrained to reduce the risk of overfitting. To further enhance transparency, SHapley Additive exPlanations (SHAP) values were employed to quantify the contribution of each cognitive and academic feature to individual predictions. The uniqueness of this approach lies in the AI-enabled integration of cognitive profiling with fine-grained behavioral data from digital learning environments, allowing cognitive characteristics and task-level performance to be examined jointly rather than in isolation. This synthesis enables the identification of individualized learning pathways that extend beyond traditional assessment practices, offering a structured yet interpretable framework for precision-oriented instructional design.

To assess the robustness of the machine learning models under these constraints, validation was performed using a Leave-One-Out Cross-Validation (LOOCV) strategy, which is particularly suitable for small datasets. This approach ensured that each student's data was used once as a test case while the remaining samples were used for training, maximizing data utilization and reducing estimation bias. No resampling or class-balancing techniques were applied, as class distributions were monitored and remained sufficiently balanced for exploratory analysis. Model evaluation emphasized the stability of class assignments and the consistency of feature importance across validation folds, rather than reliance on single-point accuracy, F1-Scores, or confusion matrices, which can be unstable and potentially misleading in small-sample settings. Given the exploratory and precision-oriented nature of the study, emphasis was placed on effect sizes, correlation coefficients, and interpretable feature contributions instead of inferential significance testing alone. This validation strategy strengthens confidence in the observed patterns while maintaining methodological rigor appropriate to the scale and aims of the study.

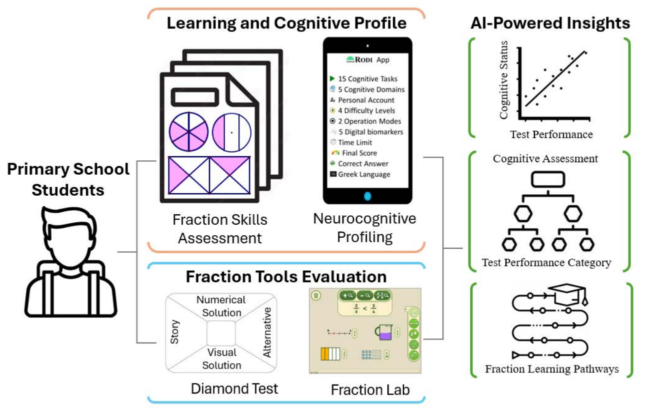

Fig. 1. Overview of the study methodology linking cognitive profiling, fraction performance assessment, and AI-based analysis to identify individualized learning pathways.

Students were classified into meaningful cognitive profiles by examining both their cognitive abilities and their performance in tasks assessing conceptual and procedural knowledge. Using validated research instruments, this classification sheds light on underlying learning patterns and informs the design of personalized instructional pathways. Table 1 presents a structured mapping of these cognitive profiles to adaptive learning pathways, linking key RODI indicators (Visual Category and Attention/Executive Function) with levels of Conceptual Understanding. Based on these combinations, distinct instructional trajectories are proposed, each tailored to the learner's cognitive strengths and needs. For example, students demonstrating strong visual processing and conceptual understanding are directed toward more autonomous and exploratory pathways, while those with lower cognitive skill levels are supported through scaffolded or manipulative-based interventions. The table illustrates how different profiles correspond to specific sequences of instructional tools, including Fraction Lab (FL) and Diamond Paper (DP), ranging from self-directed exploration to intensive guided support. This structured alignment enables a more targeted and flexible approach to instruction, ensuring that learning activities are matched to students' cognitive readiness and learning characteristics.

The methodological framework relies on two complementary categories of assessment tools. The first captures cognitive characteristics and mental processes, providing a rich description of students' learning dispositions and potential challenges. The second measures academic performance through structured digital tasks that require both conceptual reasoning and procedural execution. Together, these assessments form a comprehensive approach that links cognitive traits with academic outcomes, enabling a clearer identification of the types of support different learners may need. A flowchart of the methodology followed for the purposes of this work can be seen in Fig. 1.

Table 1. Adaptive learning pathways based on students' cognitive profiles, linking RODI indicators with conceptual understanding and attention/executive function results to suggest targeted sequences of Fraction Lab and diamond paper activities

RODI-VC	ConcU	RODI-AE	Learning Pathway	Tool Sequence
● High	● High	● High	Enrichment Track	Self-directed FL + DP exploration
● High	● High	● Low	Visual-first Track	FL only (6–8 sessions) → Optional DP for review
● High	● Low	● High	Procedure-to-Concept	DP only (5–6 sessions) → Brief FL introduction (2–3 sessions)
● High	● Low	● Low	Exploratory Track	FL primary (8–10 sessions) → Minimal DP (assessment only)
● Low	● High	● High	Concept-first Track	DP only (no FL needed) → Verbal/symbolic work
● Low	● High	● Low	Guided Conceptual	DP with heavy scaffolding → No FL (visual overload risk)
● Low	● Low	● High	Foundation Track	Physical manipulatives only → DP (4 sessions)
● Low	● Low	● Low	Intensive Scaffold	Manipulatives (10+ sessions) → DP (modified)

Note: ● High ● Low

A. Participants and Procedure

In Greece, the curriculum for fractions is generally completed by the end of sixth grade in primary school. More advanced topics, such as complex fractions, are introduced in the first year of middle school, with an emphasis primarily on procedural knowledge. Previous research within the Greek educational system has highlighted persistent individual differences in students' mathematical skills from the first to the third year of middle school [23]. This observation raises critical questions regarding the extent of students' conceptual and procedural understanding of fractions by the end of primary education and whether these differences stem from factors such as prior experience, cognitive profiles [27], or systemic influences within the Greek educational framework.

The study involved primary school graduates (mean age = 11.7 years, SD ≈ 0.5, range: 11–12 years), with an equal distribution of male and female participants (50% female). Students were recruited from six different schools in the Municipality of Argos-Mycenae, including the 2nd, 3rd, 4th, and 6th Argos Primary Schools, Kefalari Primary School, and Maltezos Private Educational Institutions. The sample represented a diverse educational background, including both public and private school students, ensuring a broad perspective on fraction understanding at the end of primary education. Participation required written parental consent, and all testing was conducted in private sessions. During these sessions, students worked individually under the remote supervision of a teacher, who monitored the process using TeamViewer. In the final phase of the study, the teacher introduced the Fraction Lab software to each student on a computer and observed their responses to a corresponding questionnaire administered through Microsoft Forms. This setup enabled real-time tracking of students' behaviors and response patterns while interacting with the Fraction Lab.

The final part of the study involved the "Diamond paper" activity. In this section, students completed four questions on an A4 sheet of paper and were provided with markers in various colors and pencils for their responses. Each session lasted between 45 and 68 min, ensuring sufficient time for the completion of all activities and observations. All students answered the same questions in all tests; the variation in completion time depended on the time each student needed to respond.

The relatively small sample size (n = 29) reflects the exploratory and high-resolution nature of the study, which prioritizes detailed cognitive profiling and fine-grained analysis of learning behavior over population-level inference. While this limits the generalizability of the findings, it allows for the identification of meaningful individual-level patterns that may be obscured in larger but less detailed datasets. The results should therefore be interpreted as indicative of underlying cognitive-learning relationships rather than as definitive estimates, providing a foundation for future studies with larger and more diverse samples.

B. Ethics and Data Governance

The study was conducted in accordance with established ethical standards for research involving human participants and, in particular, minors. Ethical approval for the research protocol was granted by the Research Ethics and Deontology Committee of the Ionian University (Approval No. 2191/14-06-2023). All procedures followed the principles of voluntary participation, respect for participants, and data protection.

Written informed consent was obtained from the parents or legal guardians of all participating students prior to data collection. In addition, students were informed about the purpose and procedures of the study in an age-appropriate manner and were assured that their participation was voluntary and that they could withdraw at any point without any consequences. No incentives were provided, and participation or non-participation had no impact on students' school evaluation.

Special care was taken to protect participants' privacy and personal data. All collected data were pseudonymized at the point of collection using unique participant codes, and no directly identifiable personal information was stored alongside performance or cognitive data. Digital records, including behavioral logs generated by the learning tools, were collected solely for research purposes and were not used for assessment or monitoring beyond the scope of the study. Data were stored in encrypted form on secure servers with restricted access. Only the principal researcher and the supervising researchers had access to the raw data. Access control mechanisms were applied to prevent unauthorized use, and data handling complied with applicable data protection regulations. The data will be retained for a limited period for research verification purposes and will be securely deleted thereafter. Participants and their guardians were informed of their right to request data deletion at any stage.

The use of digital tools and remote supervision was limited to ensuring the smooth execution of the learning activities and data collection process. No audiovisual recordings or biometric identifiers were stored as part of the present analysis. These measures ensured that the study respected both ethical research practice and the rights and well-being of child participants.

C. Assessment Instruments and Analytical Tools

The current study employed a unique combination of tools categorized into two distinct types. The first category focused on assessing students' learning and cognitive profiles, utilizing the RODI application to provide valuable insights into their individual skills and mental processes. The second category evaluated students' academic performance, specifically their proficiency in mathematics, through the Procedural and Conceptual Understanding of Fractions Tool and the Fraction Lab. Additionally, Diamond Paper was integrated to bridge conceptual and procedural knowledge, offering a comprehensive framework for analyzing students' understanding of fractions. This methodological approach is novel and, to the best of our knowledge, has not been previously reported in educational research.

1) Learning and cognitive profile assessment tools

The RODI application was employed to evaluate the cognitive skills of the participants. Specifically designed to enhance cognitive skills and address memory-related challenges, the tool offers a wide range of tasks targeting key cognitive domains, including attention, memory, language and perceptual-motor skills [49, 50]. To accommodate the age and developmental stage of the participants, the Easy difficulty level was selected. This level provided a structured and accessible assessment environment, with answers presented in various formats. The assessment was conducted without imposing time constraints, and all data collected—such as response times, answer modifications, and accuracy rates—were securely stored in an encrypted format within Firebase Cloud Storage [22, 51].

The RODI application encompasses a comprehensive set of tasks distributed across three principal cognitive domains.

Within the Visual domain, participants completed the Shapes Task, Color-Shape Task, Table Color and Shape Task, Object Series Task (image), and New Object Task (image), all of which primarily assessed visual working memory and visuospatial processing skills. In the Learning and Memory domain, tasks such as the Who Task (image), New Object Task (word), Calculating Task, Object Series Task (word), Name Task, and Descending Integer Task targeted cognitive processes related to memory recall, information integration, and numerical reasoning. Finally, the Attention and Executive Function domain included the Back Task administered at three different delay intervals (1st, 2nd, and 3rd), along with the Who Task (name) and other activities specifically designed to evaluate attentional control, cognitive flexibility, and executive functioning. The diverse array of tasks, validated through machine learning driven feature selection and psychometric evaluation, allowed for a nuanced analysis of participants' cognitive profiles, thereby supporting the study's broader aim of investigating the relationship between cognitive functions and performance on fraction-related activities.

The Procedural and Conceptual Understanding of Fractions Tool [23] is a rigorously validated instrument, widely recognized in Greece for assessing students' knowledge of fractions. It consists of 25 tasks, designed to measure both procedural and conceptual understanding. Ten items focus on procedural skills, such as executing fraction operations, while fifteen items evaluate conceptual understanding, including tasks like estimating outcomes, comparing, and ordering fractions. To accurately target conceptual knowledge, all conceptual tasks were formulated in a multiple-choice format to prevent the reliance on procedural strategies. The tool demonstrates strong psychometric properties, with content validity established through expert review and a high Content Validity Index (CVI = 1). Internal consistency was confirmed by Cronbach's alpha values of 0.921 for procedural tasks and 0.731 for conceptual tasks. The instrument was administered via Google Forms, and success rates were analyzed separately for procedural and conceptual sections, providing a robust and comprehensive measure of students' fractional competence. The assessment instrument was administered electronically using a secure Microsoft Forms environment. Due to ethical restrictions related to child participants, direct public access to the test is limited; however, the full instrument and scoring scheme are available from the corresponding author upon reasonable request.

2) Fraction knowledge evaluation tools

Fraction Lab is an interactive and research-validated digital environment, developed by Mavrikis *et al.* [52–54] at University College London (UCL), designed to enhance both conceptual and procedural understanding of fractions. Grounded in a principled design approach and co-developed with specialist mathematics teachers, the platform combines exploratory learning activities with structured exercises to promote deeper mathematical reasoning. It enables students to visualize fractions through multiple representations, such as number lines, area models, set models, and liquid measures. Key features of Fraction Lab include dynamic partitioning of shapes, the creation of equivalent fractions, and the joining of different fractional parts, fostering a more intuitive grasp of fraction equivalence and operations. Student interactions were securely captured and stored in encrypted form, with all responses recorded using a secondary laptop provided specifically for the study [55, 56].

In the context of this research, Fraction Lab was utilized to assess students' understanding of fraction equivalence and ordering through a series of carefully designed tasks [56]. It is a publicly accessible digital learning environment[1]. A range of fraction types was included in the assessment tasks, encompassing unit fractions, reducible and irreducible fractions, proper and improper fractions, as well as like and unlike fractions.

Inspired by Boaler's framework [57, 58] and developed following recommendations by Cathy Williams, Diamond Paper tool represents a novel proposal by the authors, adapted from the widely recognized Diamond Template. It was specifically designed to align with the recommended class material and informed by the authors' extensive classroom experience. The Diamond Paper prompts students to express relationships between fractions in four distinct ways: (a) creating a narrative using the numbers (conceptual understanding), (b) illustrating the relationship with a sketch (conceptual understanding), (c) representing the fractions using objects (conceptual understanding), and (d) performing the calculation (procedural knowledge). Each student received a folded A4 sheet that, when opened, formed a diamond shape with the fraction relationship $[\frac{3}{4}:\frac{1}{2}]$ written at the center. Students recorded their responses on the paper, which was later scanned, graded, and securely stored on the researcher's computer.

The Diamond Paper responses were evaluated using a structured scoring rubric designed to capture different representational aspects of fraction understanding. The activity consists of four components: D1 (Narrative representation), D2 (Visual sketch), D3 (Object-based representation), and D4 (Mathematical operation). The first three components primarily capture aspects of conceptual understanding, as they require students to interpret and represent fraction relationships through verbal, visual, and contextual representations, while the final component reflects procedural knowledge through the correct execution of the mathematical operation. In this way, the Diamond Paper activity operationalizes the Conceptual Understanding (CU) and Procedural Knowledge (PK) framework used in the present study. Each component was scored on a scale from 0 to 2.5, resulting in a total score ranging from 0 to 10. The scoring rubric distinguished between incorrect, partially correct, and correct representations of the target fraction relationship (see Table 2). Narrative and sketch components were evaluated based on the coherence and correctness of the fraction relationship expressed in verbal and visual form, while the object representation assessed the ability to translate symbolic relationships into concrete contexts. The mathematical operation component evaluated the correct execution of the corresponding fraction calculation. This rubric-based evaluation enabled a consistent and transparent assessment of students' conceptual and procedural reasoning across the four representational modes. All Diamond Paper responses were scored by the primary researcher using the predefined rubric. The scoring criteria and evaluation framework were reviewed with the supervising professor of the study to ensure alignment with the conceptual and procedural knowledge framework used in the research.

Table 2. Scoring rubric for the Diamond Paper components

Component	Score 0	Score 1–1.5	Score 2–2.5
D1 Narrative	No meaningful story or unrelated context	Story partially reflects the fraction relationship but contains inconsistencies	Coherent narrative correctly representing the fraction relationship
D2 Sketch	No visual representation or incorrect model	Partial or incomplete visual representation	Accurate visual representation of the fraction relationship
D3 Object Representation	No valid mapping to real-world objects	Partial mapping with conceptual inconsistencies	Clear and coherent representation using real-world objects
D4 Mathematical Operation	Incorrect or missing calculation	Partially correct procedure	Correct procedure and correct result

Note: Each component was scored on a 0–2.5 scale. The total Diamond Paper score ranged from 0 to 10.

D. Model Specification and Evaluation

Decision tree classifiers were implemented using standard machine learning libraries (Python, scikit-learn) and configured to prioritize interpretability given the limited sample size. The Gini index was used as the splitting criterion. Tree depth was restricted to a maximum of three levels, and a minimum leaf size constraint was applied to ensure that terminal nodes represented a meaningful number of observations. No post-pruning procedures were applied, as the depth and leaf constraints provided sufficient regularization and prevented overly complex model structures. To enable categorical prediction, student performance scores were normalized to a 0–1 scale using min–max scaling. Based on these normalized values, performance levels were defined using fixed thresholds: scores between 0.00–0.49 were classified as low, scores between 0.50–0.75 as medium, and scores between 0.76–1.00 as high. Thresholds were applied at the assessment level to account for differences in task structure and scoring across the digital learning tools used in the study.

To prevent potential data leakage during model validation, normalization parameters were computed within each training fold of the Leave-One-Out Cross-Validation (LOOCV) procedure and subsequently applied to the corresponding test observation. This approach ensures that the test data remained fully independent from the training process. Model validation was conducted using LOOCV, allowing each observation to serve once as the test case while maximizing the use of the remaining data for model training. Minimal performance reporting is provided to enhance transparency, including a confusion matrix and a multiclass evaluation metric. Given the exploratory nature of the study and the small sample size, these metrics are interpreted cautiously and primarily serve to indicate the stability and plausibility of the resulting decision rules rather than to emphasize predictive performance.

[1] Available at: https://bit.ly/fractionslabeuropa

III. RESULTS

The results presented in this section showcase the application of AI and machine learning methodologies to analyze test performance and cognitive assessments. These analyses aim to evaluate performance in relation to cognitive status, classify student profiles based on their cognitive and learning characteristics, and identify individualized learning pathways for fractions. By employing advanced data visualization techniques, the study provides a comprehensive overview of both individual and group achievements.

Before applying the AI-based analyses, students' Conceptual Understanding (CU) and Procedural Knowledge (PK) of fractions were examined descriptively to capture the extent of their development within the study cohort. Performance on the Procedural and Conceptual Understanding of Fractions Tool showed substantial variability across students, indicating heterogeneous levels of fraction competence at the end of primary education. Descriptive statistics, including measures of central tendency and dispersion, were used to characterize relative CU and PK performance within the sample rather than to establish external proficiency thresholds. This within-sample perspective provides a baseline for interpreting subsequent analyses, allowing CU and PK development to be examined in relation to students' cognitive profiles and engagement with the digital learning tools.

The relationships between students' cognitive profile features and their performance on the two digital fraction-learning tools, Fraction Lab and Diamond Paper (DP), are illustrated through a scatterplot that contrasts cognitive indicators with task outcomes (Fig. 2). Cognitive features derived from RODI and CPKn are shown along the horizontal axis, while performance scores on the fraction tasks are displayed on the vertical axis. Color coding differentiates four feature-performance pairings: blue points represent Fraction Lab performance in relation to RODI, orange points correspond to Fraction Lab performance against CPKn, green points indicate DP performance relative to RODI, and red points capture DP performance in comparison to CPKn. This distinction enables a clear view of how each learning tool aligns with specific cognitive dimensions. A pronounced clustering of green points in the upper-right region indicates that students with higher RODI scores tend to achieve stronger performance on Diamond Paper tasks. In contrast, Fraction Lab performance is more widely distributed across the mid-to-high score range, consistent with the exploratory nature of the environment and its engagement of diverse cognitive strengths. Students positioned in the lower range of cognitive indicators generally demonstrate weaker or moderate performance across both tools, highlighting the role of cognitive readiness in shaping fraction learning outcomes.

As shown in Fig. 2, color coding denotes four feature-performance pairings: blue represents Fraction Lab performance vs. RODI, orange represents Fraction Lab vs. CPKn, green represents DP vs. RODI, and red represents DP vs. CPKn. The visualization highlights how different cognitive indicators align with students' success in each tool, revealing distinct performance patterns across cognitive profiles.

The associations between students' cognitive profile features and their performance on the two digital fraction-learning tools are summarized through a correlation matrix linking RODI and CPKn indicators with outcomes in Fraction Lab and Diamond Paper (DP) (Fig. 3). This analysis offers an initial perspective on how specific cognitive dimensions relate to students' conceptual and procedural engagement with fraction tasks. A moderate positive correlation is observed between RODI scores and Fraction Lab performance (r = 0.49), suggesting that stronger cognitive features captured by RODI support learning in the visually rich and exploratory environment of Fraction Lab. The relationships between CPKn and performance in both tools are weaker, though still positive, indicating a more indirect influence on learning outcomes. Performance in Diamond Paper exhibits only small positive correlations with both RODI and CPKn, alongside a slight negative correlation with Fraction Lab, highlighting the distinct instructional emphasis of the two tools and the different cognitive processes they engage. These correlational patterns provide a foundational understanding of how cognitive profiles relate to learning outcomes and motivate the subsequent use of machine learning methods to identify individualized learning pathways.

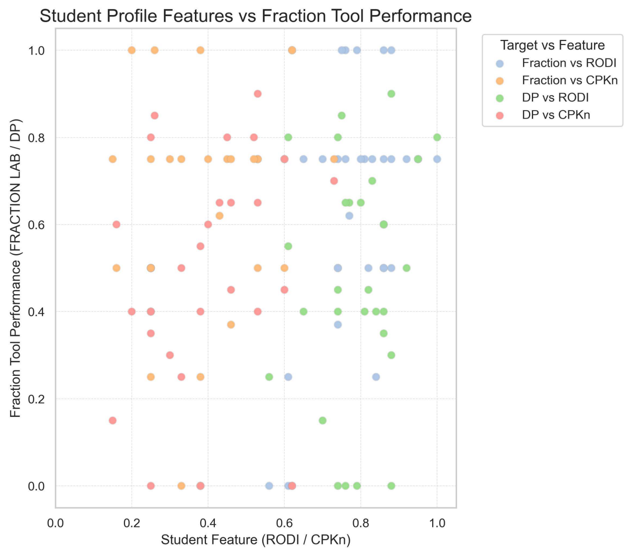

Fig. 2. Scatterplot illustrating the relationship between students' cognitive profile features (RODI and CPKn) and their performance on the two digital fraction-learning tools, Fraction Lab and Diamond Paper (DP).

The matrix (Fig. 3) highlights a moderate positive association between RODI scores and Fraction Lab performance, supporting the role of visual-conceptual processing in exploratory fraction learning, while weaker correlations with Diamond Paper reflect its stronger reliance on structured conceptual reasoning. These patterns provide empirical support for the differentiated learning pathways discussed in the Results and Conclusions sections.

To further examine patterns in student performance, decision tree classifiers were employed to categorize learners based on their scores and cognitive profiles derived from the RODI assessment and the Procedural and Conceptual Understanding of Fractions Tool (Fig. 4). Two separate models were constructed, each classifying student performance into low, medium, or high categories, with one model centered on outcomes from the Diamond Paper tasks and the other on Fraction Lab performance. In both cases, the input features consisted of aggregate scores from the remaining assessment tools, allowing performance to be interpreted within a broader cognitive and academic context. The Visual Category (VC) of the RODI application consistently emerged as the root node, underscoring its central role in the classification process. The resulting trees were intentionally shallow and structurally simple, enabling clear differentiation between performance levels while maintaining interpretability. These models illuminate how cognitive and learning profiles relate to students' engagement and success across the two fraction-learning environments.

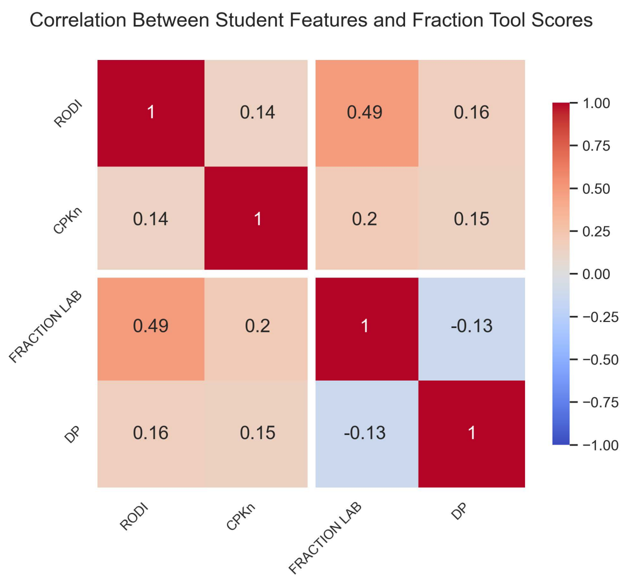

Fig. 3. Correlation matrix illustrating the relationships between students' cognitive profile features (RODI and CPKn) and their performance on the two digital fraction-learning tools, Fraction Lab and Diamond Paper (DP).

Feature importance patterns derived from the two decision tree models offer insight into the relative contribution of cognitive and academic factors in predicting student performance across the two learning tools. Gini-based importance scores were computed separately for the model predicting Diamond Paper outcomes and for the model focused on Fraction Lab performance, highlighting the dimensions most relevant for distinguishing low, medium, and high performers (Fig. 5). In the Diamond Paper model, Conceptual Understanding (ConcU) emerged as the dominant predictor, with substantially greater influence than the remaining features, reflecting the strong reliance of these tasks on conceptual reasoning and structured interpretation. Visual Category (RODI-VC), along with the RODI-LM and RODI-AE subscales, also contributed meaningfully, indicating that visual processing, logical manipulation, and attentional engagement support performance in more complementary ways. A contrasting pattern was observed in the Fraction Lab model, where RODI-VC emerged as the most influential feature by a wide margin, consistent with the visually rich and exploratory nature of the environment. ConcU remained an important secondary contributor, while the remaining cognitive indicators played more modest roles. Procedural knowledge did not exert a substantial influence in either model, suggesting that procedural fluency alone does not strongly differentiate performance within these interactive learning contexts.

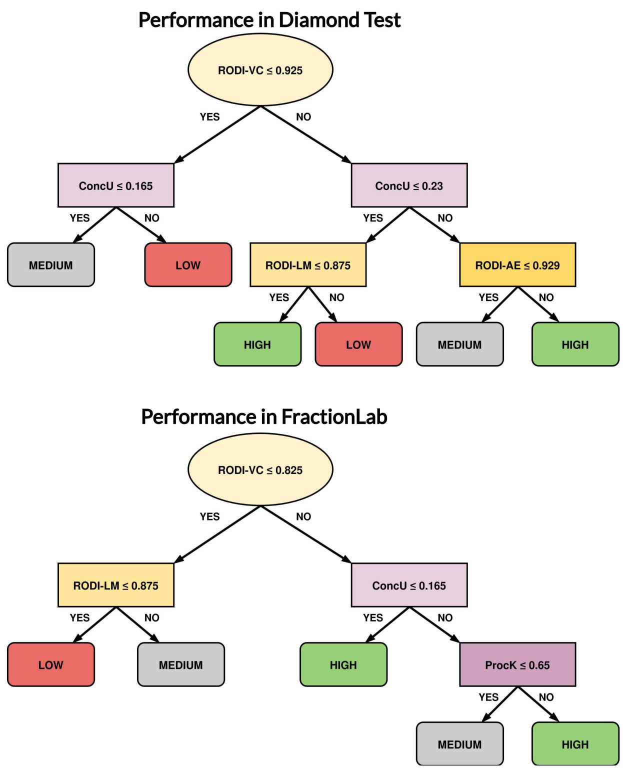

Fig. 4. Decision trees classifying students' performance as low, medium, or high based on learning profiles from RODI and the Procedural and Conceptual Understanding of Fractions Tool. The first tree focuses on Diamond Paper tasks and the second on Fraction Lab tasks. Input features include overall scores from the other three tools. In both trees, RODI's Visual Category (VC) serves as the root node, with the resulting structures effectively distinguishing between the three performance categories.

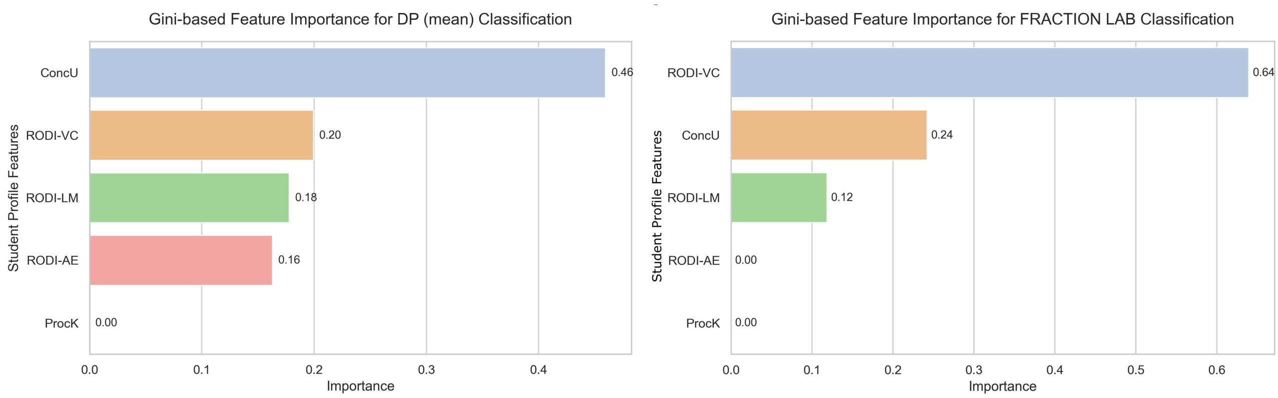

Fig. 5. Gini-based feature importance scores derived from the two decision tree classifiers predicting student performance levels in Diamond Paper (left) and Fraction Lab (right). The plots highlight the relative contribution of each cognitive and academic feature to the classification process, showing that Conceptual Understanding (ConcU) is the strongest predictor for Diamond Paper, while the Visual Category of the RODI assessment (RODI-VC) plays the dominant role in the Fraction Lab model.

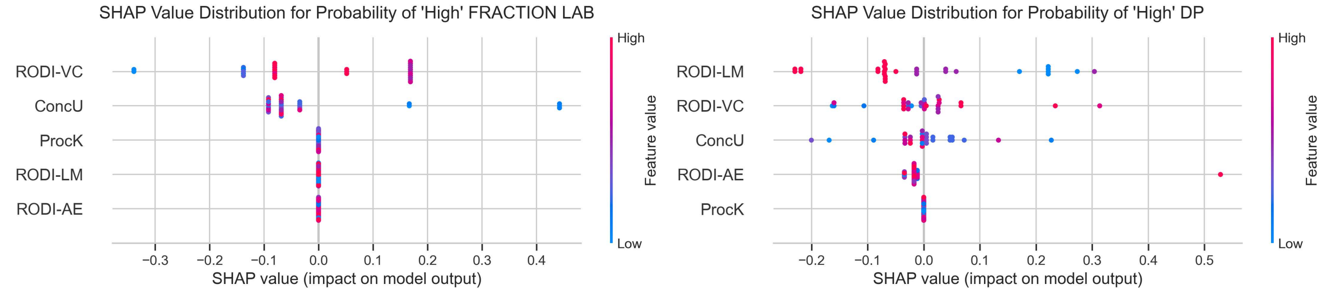

Fig. 6. SHAP value distributions for the high-performance predictions in the Fraction Lab model (left) and the Diamond Paper model (right). Each point represents a student, with SHAP values indicating the magnitude and direction of each feature's influence on the model output. The color gradient reflects the underlying feature values, where blue corresponds to lower values and pink to higher values. The visualization highlights the strong contribution of RODI-derived cognitive features, particularly visual-conceptual and logical-manipulation components, to the likelihood of achieving high performance in both learning tools.

To further enhance model interpretability, SHAP values were employed as a complementary, model-agnostic method to examine how individual features influenced predictions of high student performance (Fig. 6). This approach allows a more detailed assessment of feature contributions at the individual level, enabling verification of whether the patterns identified through the decision tree structures remain consistent under a finer-grained explanatory framework. The combined use of decision tree importance metrics and SHAP analysis strengthens the interpretability of the models and provides a more nuanced understanding of the cognitive mechanisms underlying student performance across the two learning environments.

The distribution of SHAP values reveals distinct contribution patterns for the Fraction Lab and Diamond Paper models. In the Fraction Lab model, higher values of the RODI Visual Category (RODI-VC) consistently push predictions toward the high-performance class, as reflected by the concentration of high-value points on the positive side of the SHAP axis. Lower RODI-VC values tend to exert the opposite effect, underscoring the central role of visual-conceptual processing in this exploratory environment. In contrast, the Diamond Paper model exhibits a more distributed pattern of influence, with RODI-LM and RODI-VC providing the strongest positive contributions when elevated. Conceptual Understanding also contributes positively, though to a lesser extent, while procedural knowledge remains close to neutral in both models. These patterns reinforce the conclusion that cognitive features related to visual reasoning and logical manipulation are primary drivers of strong performance, supporting the identification of individualized learning pathways grounded in cognitive profiles.

IV. Discussion

A. *Uncovering Relationships between Digital Cognitive and Fraction Knowledge Tools*

Table 3 presents the results of an analysis examining the associations between cognitive domains—Visual Perception, Complex Attention/Executive Function, and Learning and Memory—and various performance outcomes. The primary outcome variables include procedural knowledge (PK, labeled as B1) and conceptual understanding (CU, labeled as B2). Additionally, the analysis evaluates performance in Fraction Lab and explores the relationships between cognitive domains and the four components of the Diamond Paper (D1–D4), each assessing distinct aspects of procedural and cognitive skills. These findings provide insights into how specific cognitive abilities influence various types of knowledge and performance, contributing to a deeper understanding of the cognitive processes underlying learning and problem-solving. Such insights may inform the development of targeted educational interventions to enhance learning outcomes.

The data were analyzed holistically to identify cognitive profiles among students based on their performance in the RODI tasks and the results from other assessment tools. The RODI test provided valuable data for evaluating students' cognitive abilities and their relationship to academic performance. By analyzing the results across RODI's categories—Visual Perception, Attention and Executive Function, and Learning and Memory—patterns were identified that correlated with students' academic performance, offering a nuanced understanding of how these domains contribute to learning outcomes (Table 3).

Table 3. Associations between cognitive abilities and research tools

RODI'S CATEGORIES	B1	B2	FRACTION LAB	D1	D2	D3	D4
Visual perception	−0.003	0.148	0.627	0.025	0.072	0.002	0.181
Complex attention/Executive function	0.376	0.252	0.219	0.320	−0.052	0.108	0.032
Learning and memory	−0.067	−0.249	0.202	0.012	−0.166	−0.108	0.333

Visual perception showed virtually no correlation with procedural knowledge (B1, −0.003) but a weak positive correlation (0.148) with conceptual understanding (B2). A stronger positive correlation with Fraction Lab (0.627) highlights the importance of visual-spatial skills in interactive fraction tasks, supporting research that links spatial reasoning to fraction performance [59]. For the Diamond Paper components (D1–D4), correlations were generally weak, with the highest correlation appearing in D4 (0.181).

Complex attention and executive function displayed a moderate positive correlation with procedural knowledge (B1, 0.376), emphasizing the relevance of skills such as attention control and problem-solving for procedural tasks. The correlation with conceptual understanding (B2, 0.252) was weaker but still meaningful. Fraction Lab performance correlated weakly (0.219), suggesting that while attention and executive function support exploratory tasks, other factors—particularly visual-spatial ability—play a stronger role. Among the Diamond Paper components, D1 had the strongest correlation (0.320), while D2 showed a slight negative correlation (−0.052), reflecting differences in cognitive demands across tasks. Learning and memory had very weak negative correlations with procedural knowledge (B1, −0.067) and a moderate negative correlation with conceptual understanding (B2, −0.249). This suggests that while memory supports the recall of steps, it alone does not lead to flexible problem-solving or deeper understanding. A weak positive correlation with Fraction Lab (0.202) implies that memory plays a modest role in interactive tasks. Among the Diamond Paper components, D4 had the strongest positive correlation (0.333), consistent with the need for memory in tasks such as fraction division.

These findings highlight the nuanced role cognitive skills play in learning fractions. They also underscore the value of combining digital assessment tools, such as RODI and Fraction Lab, with classroom teaching practices to better identify and address student needs. For instance, visual perception skills may be especially relevant for technology-based fraction tools, while executive function and attention control are critical for procedural tasks. Learning and memory, although important for recall, are less predictive of success in tasks requiring conceptual flexibility. By leveraging these insights, educators can design more effective technology-enhanced interventions. Digital tools can be used not only for assessment but also to adapt content and provide individualized support, ensuring that both procedural knowledge and conceptual understanding are addressed in the classroom. This evidence-based approach supports the broader goal of integrating technology with innovative mathematics teaching methodologies.

B. Adaptive Learning Pathways Based on Cognitive Profiles

Based on the AI analysis and the results of the cognitive and performance assessments, a set of distinct learner profiles and corresponding adaptive learning pathways were identified. The adaptive learning pathways identified in this study should be interpreted as illustrative and developmental constructs rather than fixed or exhaustive classifications, intended to highlight emerging patterns in how cognitive profiles align with fraction learning rather than to prescribe stable learner categories. These pathways reflect the ways in which students' cognitive strengths, visual-processing skills and conceptual or procedural understanding shape their engagement with Fraction Lab and Diamond Paper. Interdisciplinary curricula that integrate mathematics with art or science [60], along with activities such as competitions and game-based problem solving [61–63], have been shown to sustain students' mathematical development. The present findings extend this perspective by demonstrating how cognitive profiling can further refine the alignment between instructional design and individual learning needs.

An important limitation of this study is the relatively small sample size (n = 29), which was shaped by the depth of cognitive profiling and the use of multiple assessment modalities. Although this sample size restricts the generalizability of the findings to broader student populations, it enabled a detailed, individual-level examination of cognitive characteristics and learning behaviors that would be difficult to achieve in larger-scale studies. The analytical focus was therefore placed on identifying meaningful patterns and relationships rather than on making population-level statistical inferences. Consequently, the results should be interpreted as exploratory and hypothesis-generating, providing insights into how specific cognitive profiles align with fraction learning performance. Future research involving larger and more diverse samples is needed to validate these patterns, assess their stability across educational contexts, and examine the extent to which the identified learning pathways can be generalized and implemented at scale.

The first pathway concerns students with a visual-dominant cognitive profile, characterized by a RODI Visual Category score greater than 0.825. The AI models consistently indicated that these learners benefit most from beginning their instruction with Fraction Lab. This environment offers multiple visual representations, such as number lines, area models and set models, which appear to resonate strongly with their perceptual strengths. The correlation between RODI-VC and Fraction Lab performance, which reaches 0.627, reinforces the conclusion that visually oriented learners are able to convert perceptual clarity into conceptual and procedural gains. For this group, instruction may begin with 2–3 Fraction Lab sessions focused on visual representations (e.g., area models for fraction comparison, addition, and subtraction), followed by a transition to Diamond Paper activities once conceptual stability is observed.

A second pathway emerged for learners with strong conceptual understanding and solid executive-function skills, reflected in ConcU scores above 0.165 and supportive attention indicators from RODI. These students performed particularly well on Diamond Paper tasks, which require the coordination of storytelling, visualization and procedural calculation. The decision tree analysis identified ConcU as the key predictor of high Diamond Paper performance, indicating that these learners are well positioned to integrate conceptual and procedural reasoning. In this sense, Diamond Paper functions not as a remedial tool but as a cognitively demanding learning environment that challenges students with established conceptual understanding to reorganize, extend, and integrate their knowledge. For these learners, the tool supports deeper reflection and conceptual coherence by requiring the coordination of narrative, visual, and symbolic representations. Such use allows educators to purposefully extend students' thinking beyond procedural correctness toward more comprehensive and flexible fraction understanding.

A third pathway was identified for students with lower levels of visual perception. Learners whose RODI-VC scores are at or below 0.825 often require structured guidance before engaging with visually complex or exploratory tasks. The decision tree classifier consistently assigned these students to the low-performance category when visual demands increased, suggesting the need for a scaffolded design. These learners benefit from memory-based learning, explicit structuring and concrete manipulatives. For this group, Diamond Paper also serves as a diagnostic and supportive tool, enabling teachers to identify specific points of difficulty by examining how students approach the same fraction relationship through multiple representational modes. By analyzing students' narratives, visual sketches, object-based representations, and calculations, teachers can gain insight into which aspects of the numerical relationship remain fragile or misunderstood. This multidimensional perspective allows task complexity and instructional support to be adjusted in a targeted manner, supporting gradual progression toward more visually demanding and exploratory environments. Progression toward Fraction Lab should be gradual and responsive, ensuring that visual demands are introduced only once foundational understanding has been sufficiently established.

Finally, a fourth pathway corresponds to students with balanced cognitive profiles, characterized by mid-range scores across the RODI and CPKn indicators. For this group, an integrated instructional approach is most effective. Alternating between Fraction Lab and Diamond Paper allows these learners to engage both exploratory and structured forms of reasoning. The SHAP analysis showed that procedural knowledge influences high performance in Diamond Paper tasks for this group, indicating that the instructional design should remain responsive to conceptual and procedural gaps. A blended strategy that incorporates both tools in a flexible manner is therefore recommended for these students. These adaptive learning pathways demonstrate how AI-driven cognitive profiling can inform differentiated instruction, enabling a more precise match between pedagogical approaches and learners' cognitive characteristics.

V. Conclusions

The findings of this study highlight the transformative role that precision education can play in personalizing mathematics learning based on students' cognitive and academic characteristics. Through the combined use of the RODI cognitive profiling application, the Procedural and Conceptual Understanding of Fractions Tool, Fraction Lab and Diamond Paper, the study demonstrated how individualized assessment data can inform instructional decisions in meaningful ways. The identification of distinct cognitive profiles allowed for a clear understanding of how students differ in their conceptual and procedural development of fractions, revealing the need for adaptive instructional strategies that respond to these variations.

A central contribution of the study lies in the application of artificial intelligence for the analysis of complex performance and cognitive datasets. Machine learning algorithms revealed patterns that would have been difficult to detect through conventional methods. These patterns provided insight into the specific cognitive mechanisms that support learning in each environment. The strong influence of visual-conceptual processing on success in Fraction Lab, and the importance of executive function and conceptual understanding for effective engagement with Diamond Paper, illustrate how different tools align with different learner profiles. The AI-driven analyses, including decision trees and SHAP explanations, enabled the construction of four adaptive learning pathways, each attuned to a particular cognitive pattern. These pathways offer a structured approach for aligning instructional design with student needs, whether through visual-first exploration, conceptually grounded sequencing, scaffolded guidance for learners with low visual perception or blended integration for students with balanced profiles.

Certain limitations should be considered when interpreting these results. The study involved twenty-nine students, a sample size that enabled detailed cognitive and behavioral analysis but may limit the generalizability of the findings. This sample size was partly influenced by the broader research design, which included the concurrent collection of EEG data requiring specialized equipment and careful experimental procedures. However, the EEG data were analyzed in a separate study that has been published elsewhere and are not included in the analyses presented in this article. The present study focuses exclusively on cognitive profiling and performance data derived from the digital assessment tools. Future work will expand the dataset and explore how neural activity relates to cognitive profiles and fraction learning. Another limitation concerns the cross-sectional design, which captures learning at a single point in time. Longitudinal research would help clarify how personalized learning pathways influence fraction understanding over longer periods and whether improvements persist as mathematical content becomes more advanced.

Despite these constraints, the study provides a valuable contribution to the growing body of research on adaptive mathematics education. The integration of AI, cognitive profiling and digital learning tools demonstrates how technology can assist teachers by offering actionable information that supports targeted intervention. The adaptive pathways derived from this analysis show the potential of data-informed instruction to bridge learning gaps and strengthen conceptual and procedural understanding. These insights create opportunities for educators to design more responsive learning environments and to support students with diverse learning needs more effectively.

The work presented here underscores the importance of merging advanced analytical techniques with pedagogical expertise. By demonstrating how AI-enhanced precision education can inform teaching decisions in foundational areas such as fractions, the study contributes to the broader effort to integrate innovative digital solutions into mathematics education. It suggests that when cognitive profiling and adaptive technologies are combined with thoughtful instructional design, mathematics learning can become more equitable, more effective and more deeply connected to the strengths and challenges of each learner.

Informed Consent Statement

Informed consent was obtained from all subjects involved in the study.

Data Availability Statement

The data presented in this study are available on request from the corresponding author.

Conflict of Interest

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

E.L. and S.D. conceived and designed the study. E.L., A.V., and S.D. developed the research methodology. K.L. developed the software used in the study. E.L. conducted the investigation, collected and curated the data, and performed the formal analysis. A.V., S.D., and P.V. validated the methodology and the results. S.D. and P.V. provided the research resources and infrastructure. E.L., K.L., and A.V. prepared the original draft of the manuscript. S.D. and P.V. critically reviewed and edited the manuscript. S.D. and P.V. supervised the research and coordinated the project administration. All authors have read and agreed to the published version of the manuscript.

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